Virtual Reality & Physically-Based Simulation

Collision Detection

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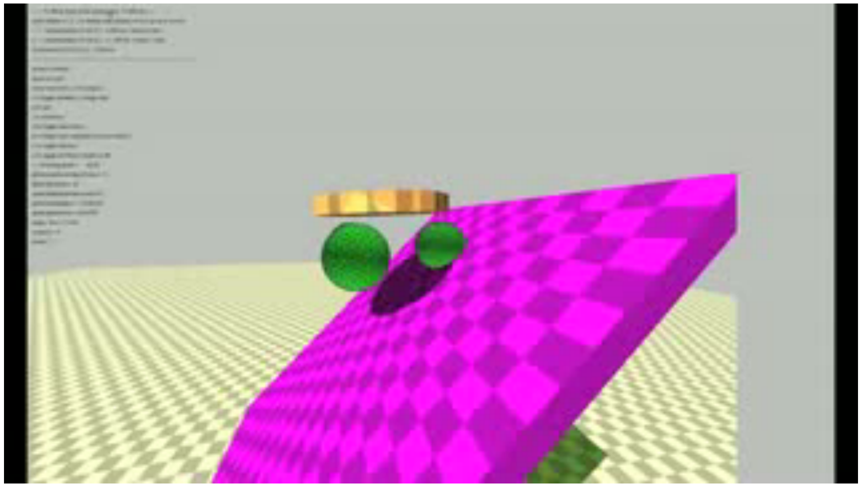
Examples of Applications

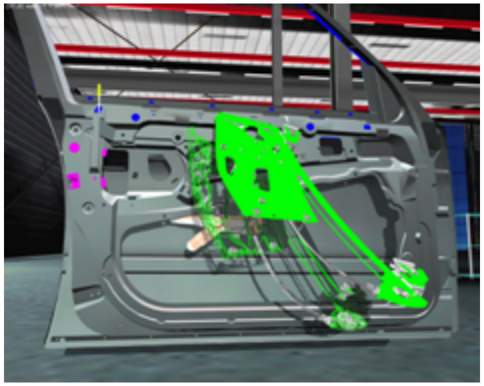
Virtual Prototyping

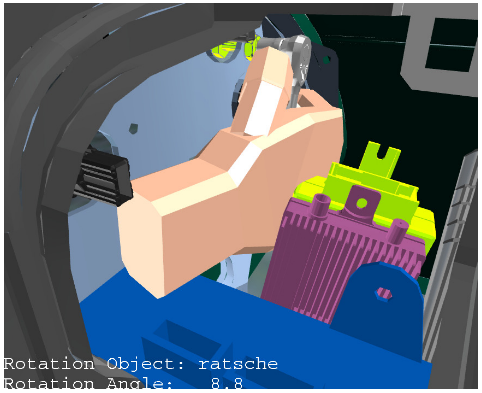


Physically-based simulation

Application Areas for Collision Detection

- Collision detection is an enabling technology for:
  - Physically-based simulation
  - Interaction in VR
  - Haptic rendering
- Application areas:
  - Games, animation, surgery, virtual prototyping, path planning, online robot collision avoidance





Collision Detection Within Simulations

- Main loop:

  Move objects

  Check collisions

  Handle collisions (e.g., compute penalty forces)

- Collisions pose two different problems:
  1. Collision detection
  2. Collision handling

- In this chapter: only collision detection

Definitions

- Given $P, Q \subseteq \mathbb{R}^3$
- The detection problem:

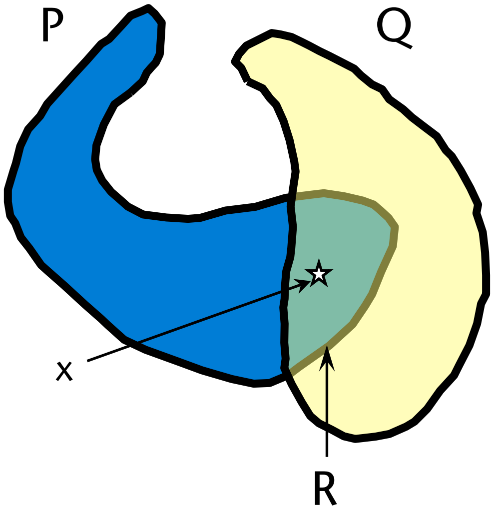
  "P and Q collide" $:\Leftrightarrow$

  $P \cap Q \neq \varnothing \Leftrightarrow$

  $\exists x \in^3 : x \in P \wedge x \in Q$

- The construction problem:

  compute $R := P \cap Q$



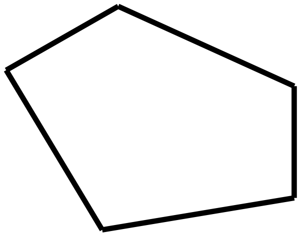
- For polygonal objects we define collisions as follows:

  P,Q collide $\Leftrightarrow \quad \exists f \in F^P \exists f' \in F^Q : f \cap f' \neq \varnothing$

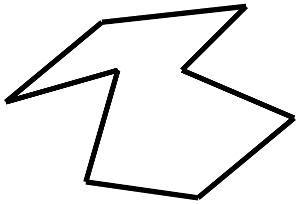
- The games community often has a different definition of "collision"

Objekt-Klassen

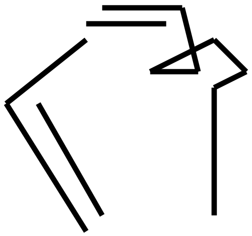
- Convex
- Closed and simple
  (no self-penetrations)
- Polygon soups
  - Not necessarily closed
  - Duplicate polygons
  - Coplanar polygons
  - Self-penetrations
  - Degenerate cardigans
  - Holes
- Deformable



konvex

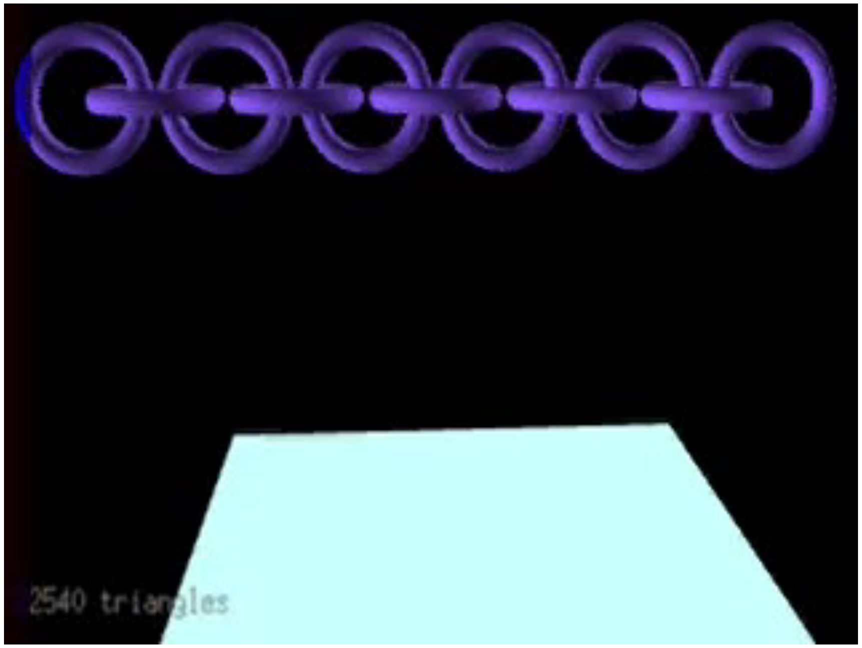


einfach & geschlossen

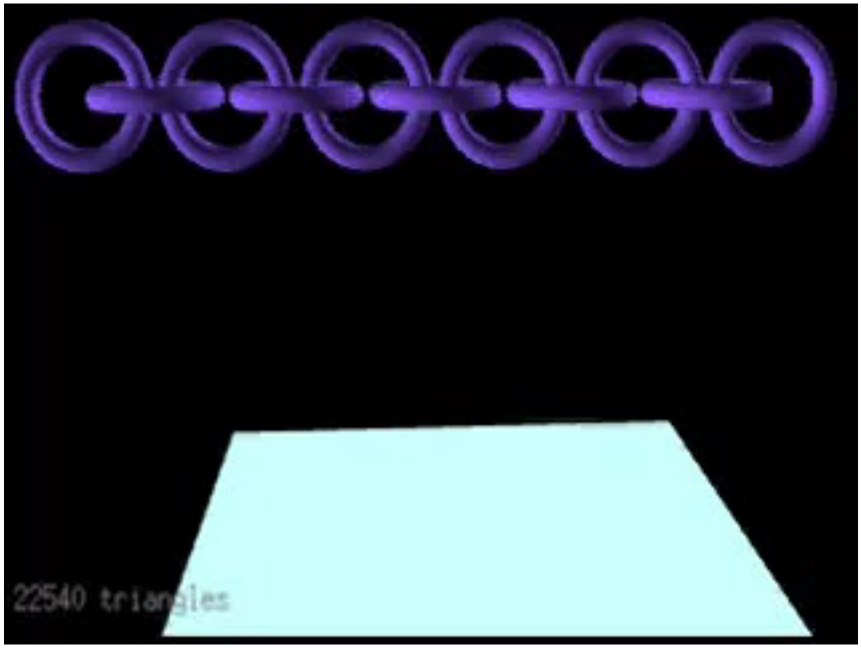


polygon soup

Importance of the Performance of Collision Detection



naïve algorithm
(test all pairs of polygons)



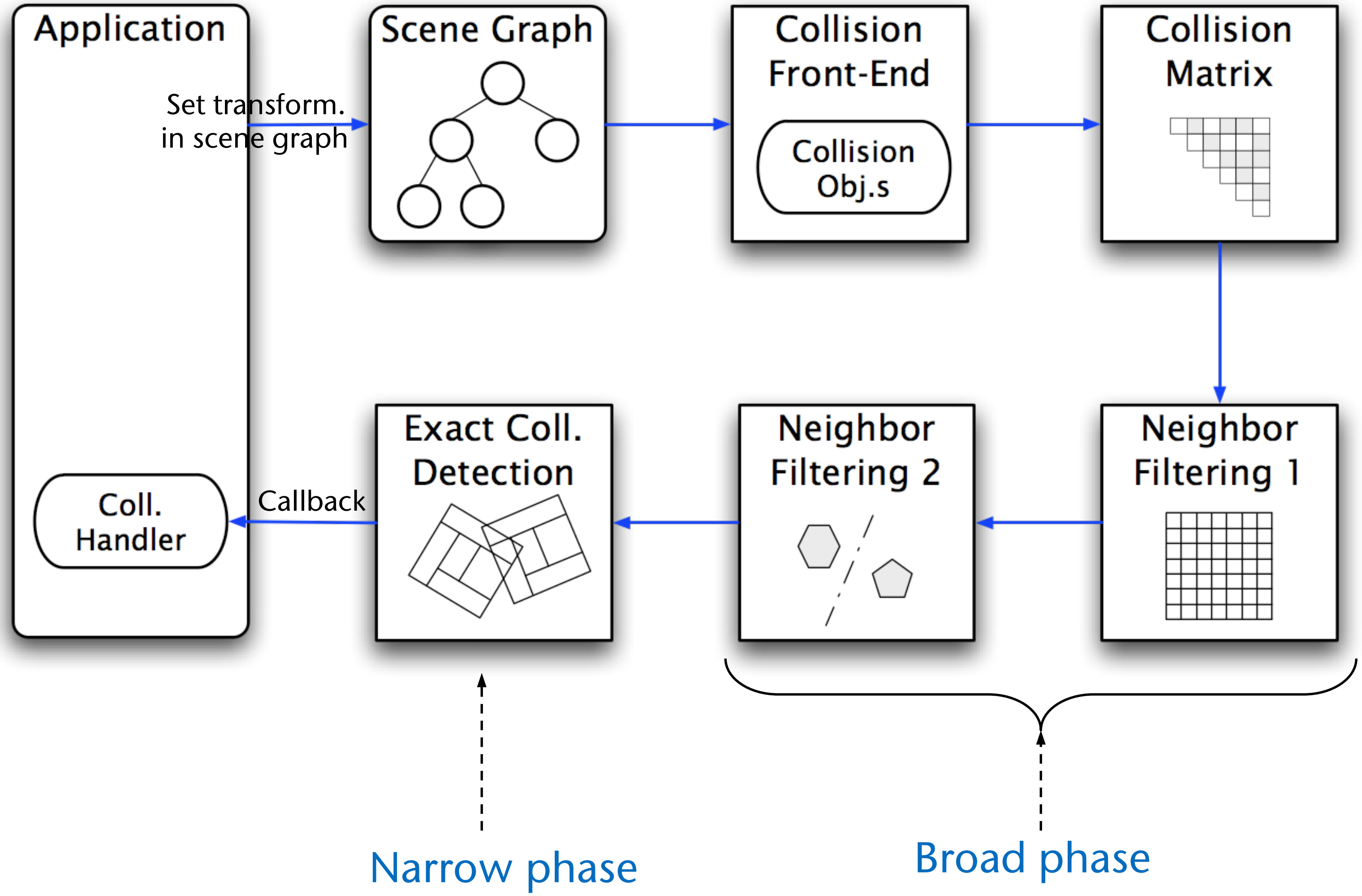
clever algorithm
(use bbox hierarchy)

Conclusion: the performance of the algorithm for collision detection determines (often) the overall performance of the simulation!

Requirements on Collision Detection

- Handle a large class of objects
- Lots of moving objects (some 1000)
- Very high performance, so that a physically-based simulation can do many iterations per frame (at least 2x 100,000 polygons in <1 millisec)
- Return a contact point ("witness") in case of collision
  - Optionally: return *all* intersection points
- Auxiliary data structures should not be to a large zu große zusätzliche Datenstrukturen (<2x);
  - Preprocessing for these auxiliary data structures should not take too long, so that it can be done at startup time (< 5sec / object)

The Collision Detection Pipeline



The Collision Interest Matrix

- Interest in collisions is specific to different applications/modules:
  - Not all modules in an application are interested in all possible collisions;
  - Some pairs of objects collide all the time, some can never collide;
- Goal: prevent unnecessary collision tests

  ⇒ Collision Interest Matrix

Obj	1	2	3	4	5	6	7	8
1			x	x	x	x		
2						x		
3						x		x
4								x
5								x
6								x
7								x
8								

- The elements in this matrix comprise:
  - Flag for collision detection
  - Additional info that needs to be stored from frame to frame for each pair for certain algorithms (e.g., the separating plane)
  - *Callbacks* in die Module

Methods for the Broad Phase

- Broad phase = one or more filtering step
  - Goal: quickly filter pairs of objects that cannot intersect because they are *too far away* from each other
- Standard approach:
  - Enclose each object within a bounding box (bbox)
  - Compare the 2 bboxes for a given pair of objects
- Assumption: n objects are moving

➢ *Brute-force* method needs to compare $O(n^2)$ bboxes

- Idea: try to determine neighbors (i.e., close objects) very quickly

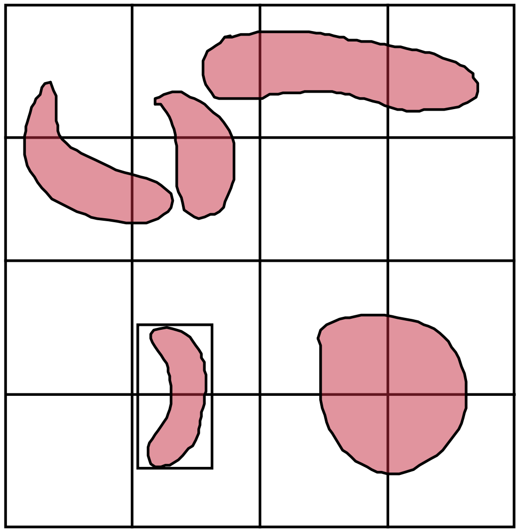
➢ 3D grid, sweep plane, etc.

The 3D Grid

Idea:

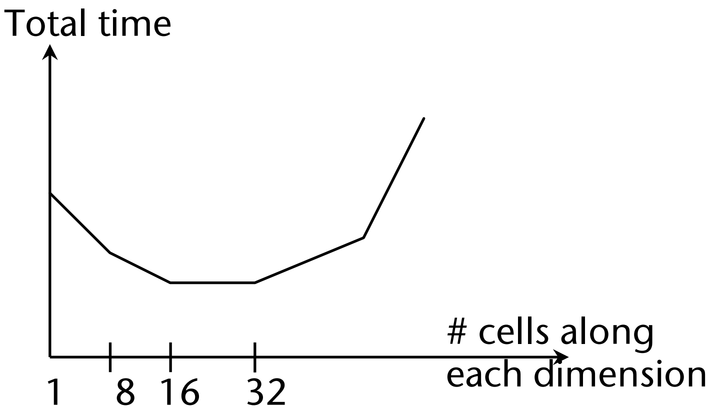
1. Partition the "universe" by a grid
2. Objects are considered neighbors, if they occupy the same cell
3. Determine cell occupancy by bbox
4. When objects move → update grid

- Neighbor-finding = find all cells that contain more than one bbox
  - Data structure here: hash table (!)
  - Collision in hash table → probably neighbor



The trade-off:

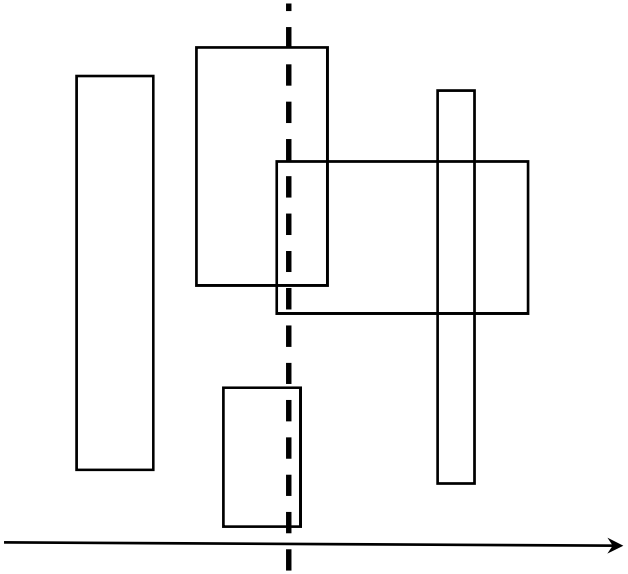
- Fewer cells = larger cells
  - ➢ Distant objects are still "neighbors"
- More cells = smaller cells
  - ➢ Objects occupy more cells
  - ➢ Effort for updating increases



The Plane Sweep Technique (Sweep and Prune)

- The idea:
  sweep plane through space
  perpendicular to the X axis
- The algorithm:

```
sort the X coordinates of all boxes
start with the leftmost box
keep a list of active boxes
jump from box border to box border:
   if current box border is the left side (= "opening"):
      check this box against all boxes in the active list
      add this box to the list of active boxes
   else (= "closing"):
      remove this box from the list of active boxes
```



Frame-to-Frame Coherence

- Observation:

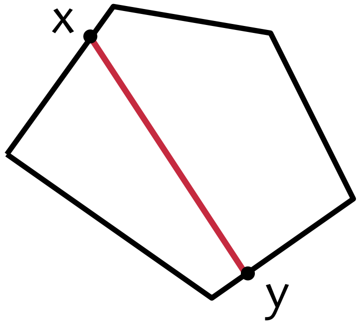
  Two consecutive images in a sequence differ only by very little (usually).

- ➢ Terminology: frame-to-frame or temporal coherence
- Examples:
  - Motion of a camera
  - Motion of objects in a film / animation
- Applications:
  - Computer Vision (e.g. tracking of markers)
  - MPEG
  - Collision detection
  - Ray-tracing of animations (e.g. using kinetic data structures)
- Algorithms based on frame-to-frame coherence are called "incremental", sometimes "dynamic" or "online" (the latter is actually the wrong term)

Convex Objects

- Definition of "convex polyhedron":

$$P \subset \mathbb{R}^3 \ \text{convex} \quad \Leftrightarrow$$
$$\forall x, y \in P : \overline{xy} \subset P \Leftrightarrow$$
$$P = \bigcap_{i=1\ldots n} H_i \quad , H_i = \text{half-spaces}$$

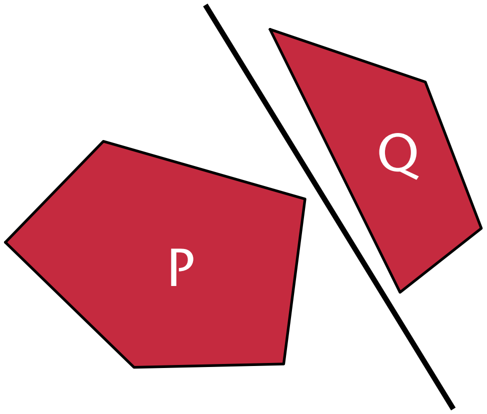


- A condition for "non-collision":

  P and Q are "linearly separable" $\Leftrightarrow$

  $$\exists \text{ half-space } H : P \subseteq H \wedge Q \subseteq H^c$$

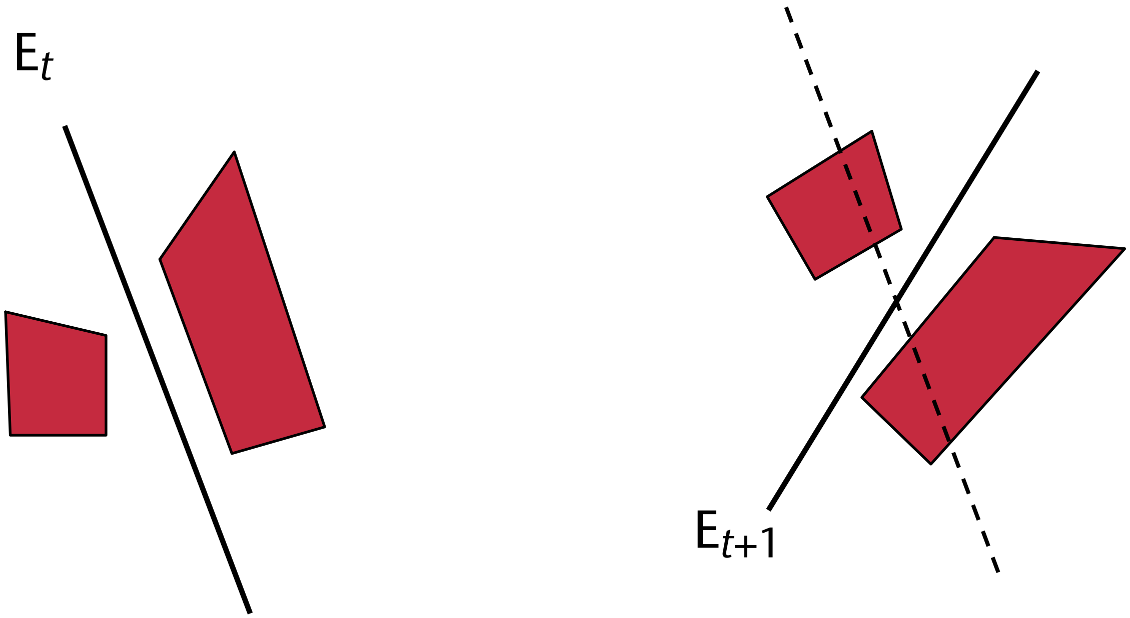
  ("P is completely to one side of H,
  Q completely on the other side")



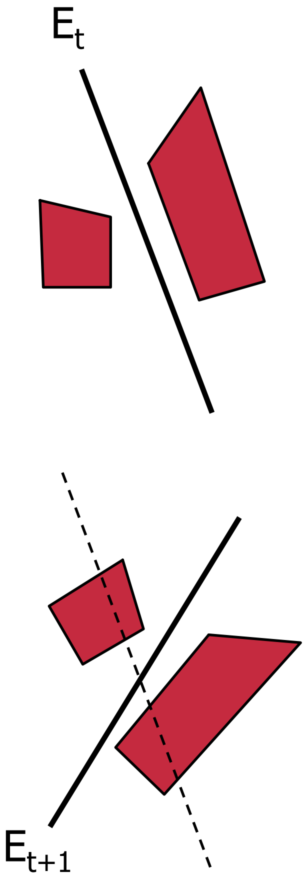
Separating plane H

The Algorithm "Separating Planes"

- The idea: utilize temporal coherence →
  if E_t was a separating plane between *P* and *Q* at time *t*, then the new separating plane E_{t+1} is probably not very "far" from E_t (perhaps it is even the same)



```
load $E_t$ = separating plane between $P$ & $Q$ at time $t$
$E := E_t$
repeat max $n$ times
    if exists $v \in \text{vertices}(P)$ on the back side of E:
        rot./transl. E such that $v$ is now on the front side of E
    if exists $v \in \text{vertices}(Q)$ on the front side of E:
        rot./transl. E such that $v$ is now on the back side of E
    if there are no vertices on the "wrong" side of E, resp.:
        return "no collision"
if there are still vertices on the "wrong" side of E:
    return "collision"   {could be wrong}
save $E_{t+1} := E$   for the next frame
```



For details on the "rot./transl. E" step → see perceptron learning algorithm

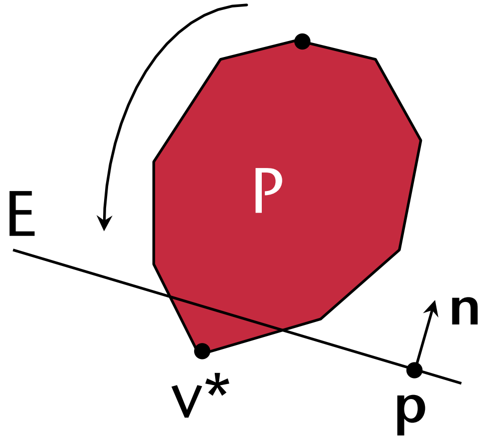
How to Find a Vertex on the "Wrong" Side *Quickly*

- The brute-force method:

  test all **v** whether $f(\mathbf{v}) = (\mathbf{v} - \mathbf{p})\cdot\mathbf{n} > 0$

- Observation:

  1. *f* is linear,
  2. P is convex ⇒ *f*(*x*) has (usually) exactly one minimum over all points *x* on the surface of P
  3. $\exists^1\, \mathbf{v}^* \;:\; f(\mathbf{v}^*) = \min$

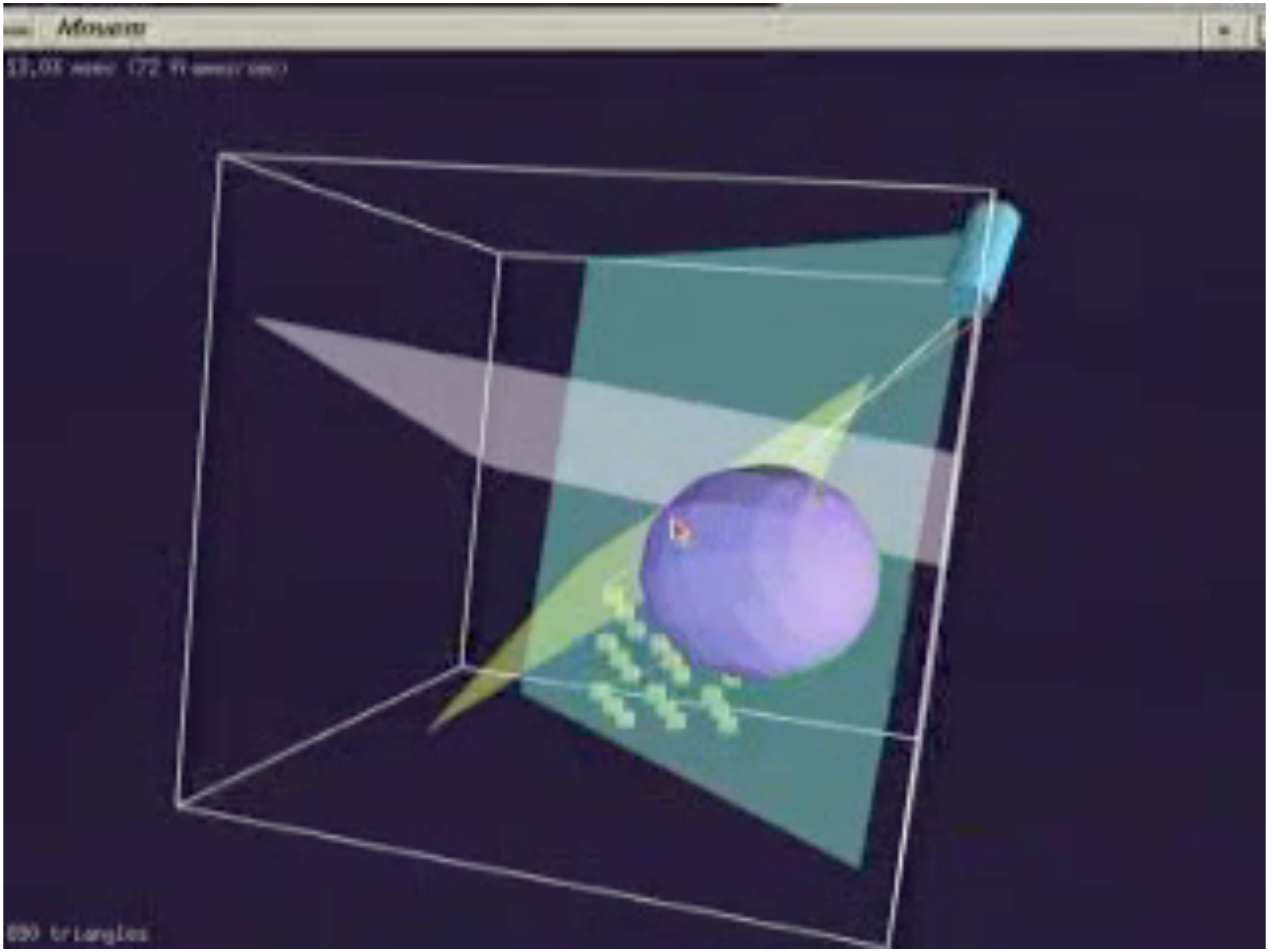


- The algorithm (steepest descent on the surface w.r.t. *f*):
  - Start with an arbitrary vertex **v**
  - Walk to the neighbor **v'** of **v** for which $f(\mathbf{v}') = \min.$ (among all neighbors)
  - Stop if there is no neighbor **v'** of **v** for which $f(\mathbf{v}') < f(\mathbf{v})$

Properties of this Algorithm

- \+ Expected running time is in O(1)!
  The algo exploits *frame-to-frame coherence*:
  if the objects move only very little, then the algo just checks whether the old separating plane is still a separating plane;
  if the separating plane has to be moved, then the algo is often finished after a few iterations.
- \+ Works even for deformable objects, so long as they stay convex
- – Works only for convex objects
- – Could return the wrong answer if P and Q are extremely close but not intersecting (bias)
- ■ *Research question: can you find an un-biased (deterministic) variant?*

Visualization



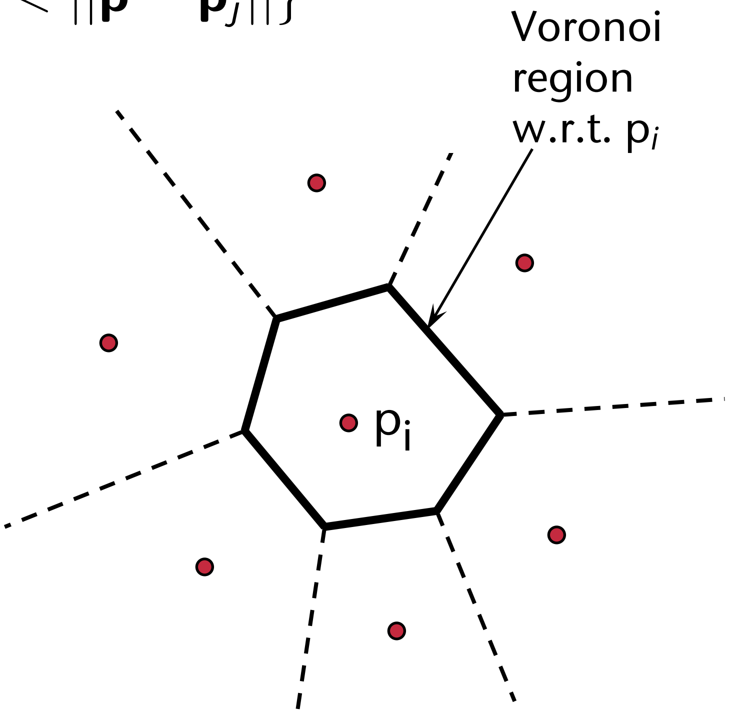
Closest Feature Tracking

- Proposed by Lin & Canny in 1992 (→ "Lin-Canny-Algorithm")
- Idea:
    - Maintain the minimal distance between a pair of objects
    - Which is realized by one point on the surface of each object
    - If the objects move continuously, then those points move continuously on the surface of their objects
- The algorithm is based on the following methods:
    - Voronoi diagrams
    - The “closest features” lemma

Voronoi Diagrams for Point Sets

- Given a set of points $S = \{\mathbf{p}_i\}$, called sites (or generators)
- Definition of a Voronoi region/cell :

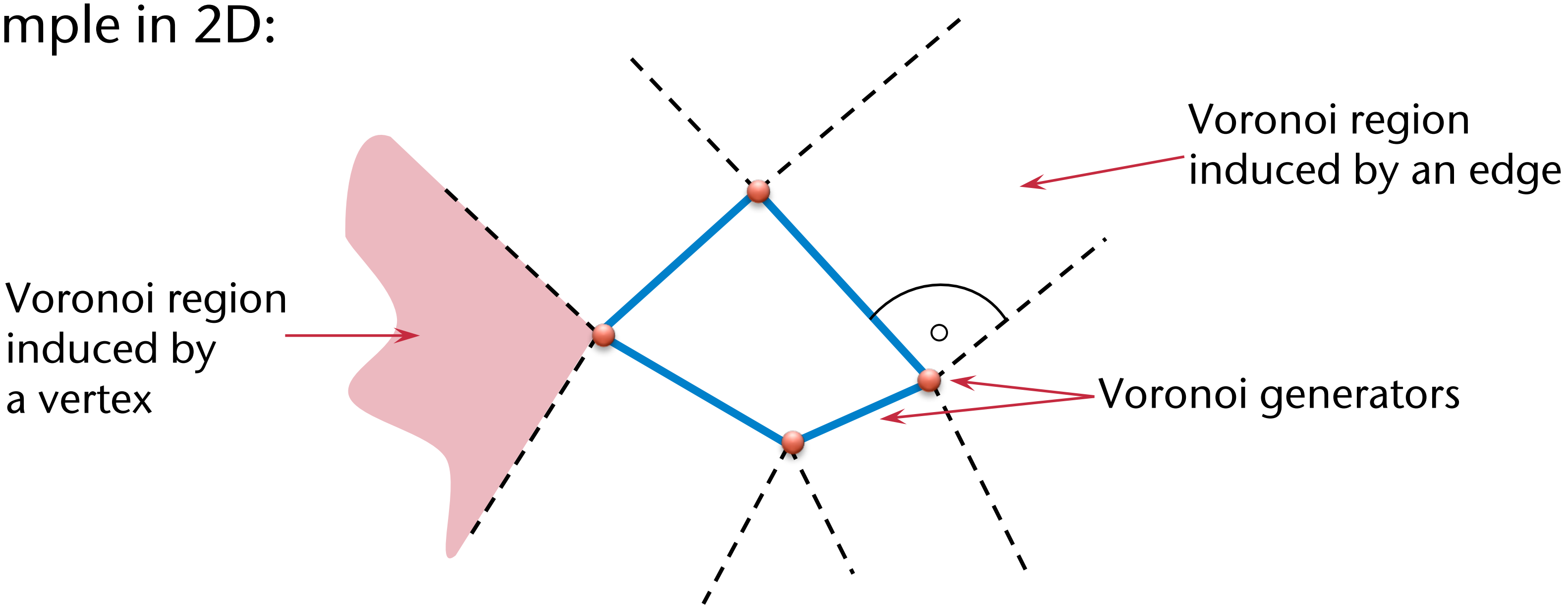
$$V(p_i) := \{\mathbf{p} \in \mathbb{R}^2 \mid \forall j \neq i : ||\mathbf{p} - \mathbf{p}_i|| < ||\mathbf{p} - \mathbf{p}_j||\}$$



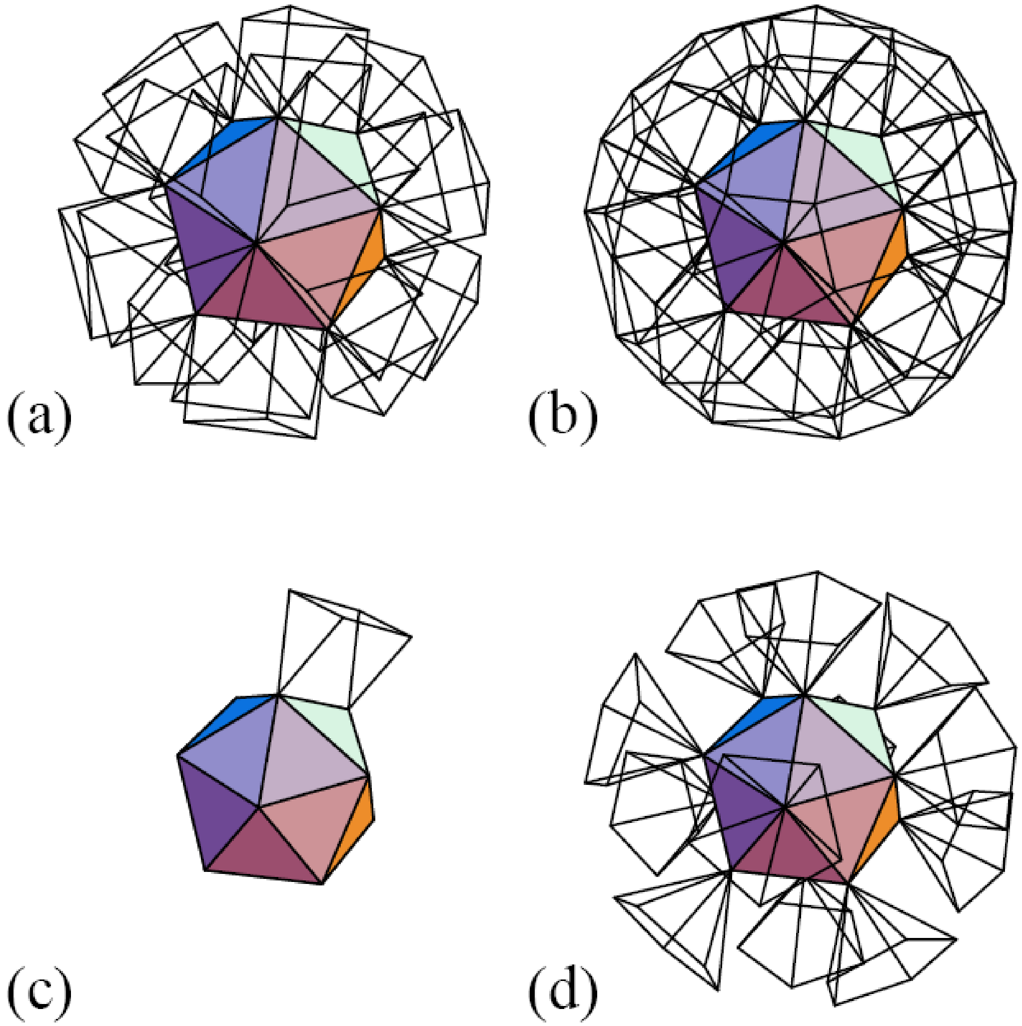
- Definition of Voronoi diagrams:
  The Voronoi diagram $\mathcal{VD}(S)$ over a set of points S is the union of all Voronoi regions over the points in S.
- $\mathcal{VD}(S)$ induces a partition of the plane into Voronoi edges, Voronoi nodes, and Voronoi regions
- Interaktive Demo: http://web.cs.uni-bonn.de/I/GeomLab/VoroGlide/

Voronoi Diagrams over Sets of Points, Edges, Polygons

- Voronoi diagrams can be defined analogously in 3D (and higher dimensions)
- What if the generators are not points but edges / polygons?
- Definition of a Voronoi cell is still the same:
  The Voronoi region of an edge/polygon := all points in space that are closer to "their" generator than to any other
- Example in 2D:



Outer Voronoi Regions Generated by a Polyhedron



The external Voronoi regions of …

(a) faces
(b) edges
(c) a single edge
(d) vertices

Outer Voronoi regions for convex polyhedra can be constructed very easily!
(We won't need inner Voronoi regions.)

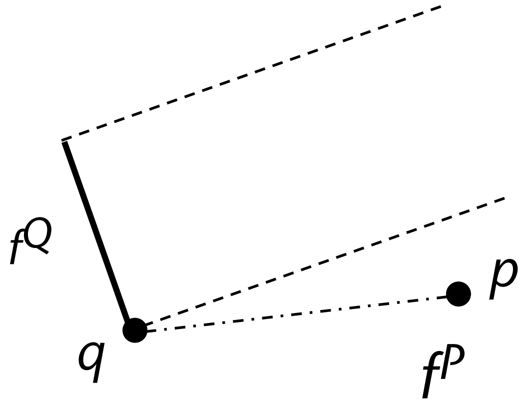
Closest Features

- Definition *Feature* f^P := a vertex, edge, polygon of polyhedron P.
- Definition "Closest Feature":
  Let f^P and f^Q be two features on polyhedra P and Q, resp., and let p, q be points on f^P and f^Q, resp., that realize the minimal distance between P and Q, i.e.

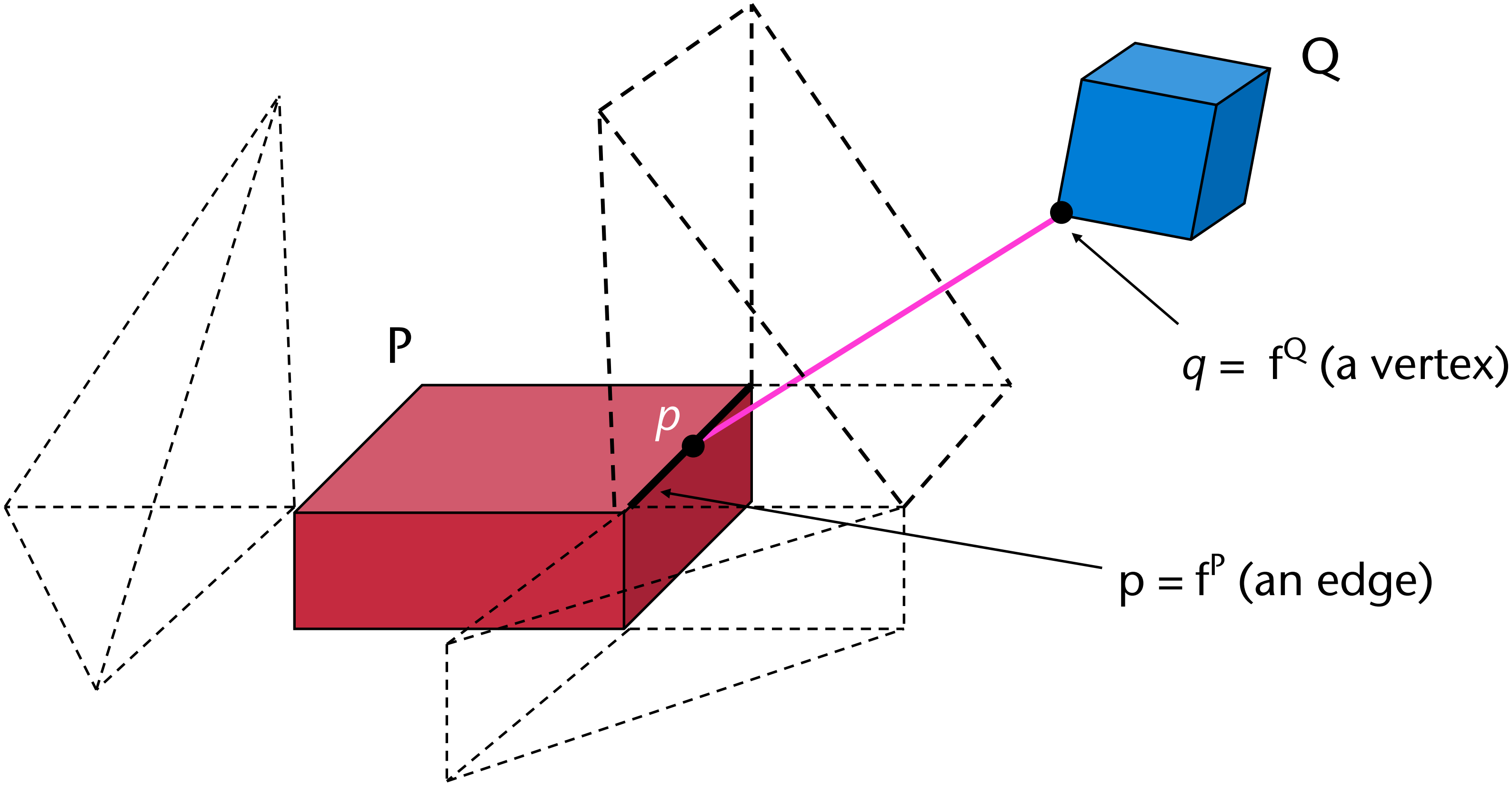
  $$d(P,Q) = d\left(f^P, f^Q\right) = ||p-q||$$

  Then f^P and f^Q are called "closest features".
- The "closest feature" lemma:
  Let $V(f)$ denote the Voronoi region generated by feature f; let p and q be points on the surface of P and Q realizing the minimal distance. Then

  f^P, f^Q are closest features ⇔ p is in $V(f^Q)$, q is in $V(f^P)$.



Example



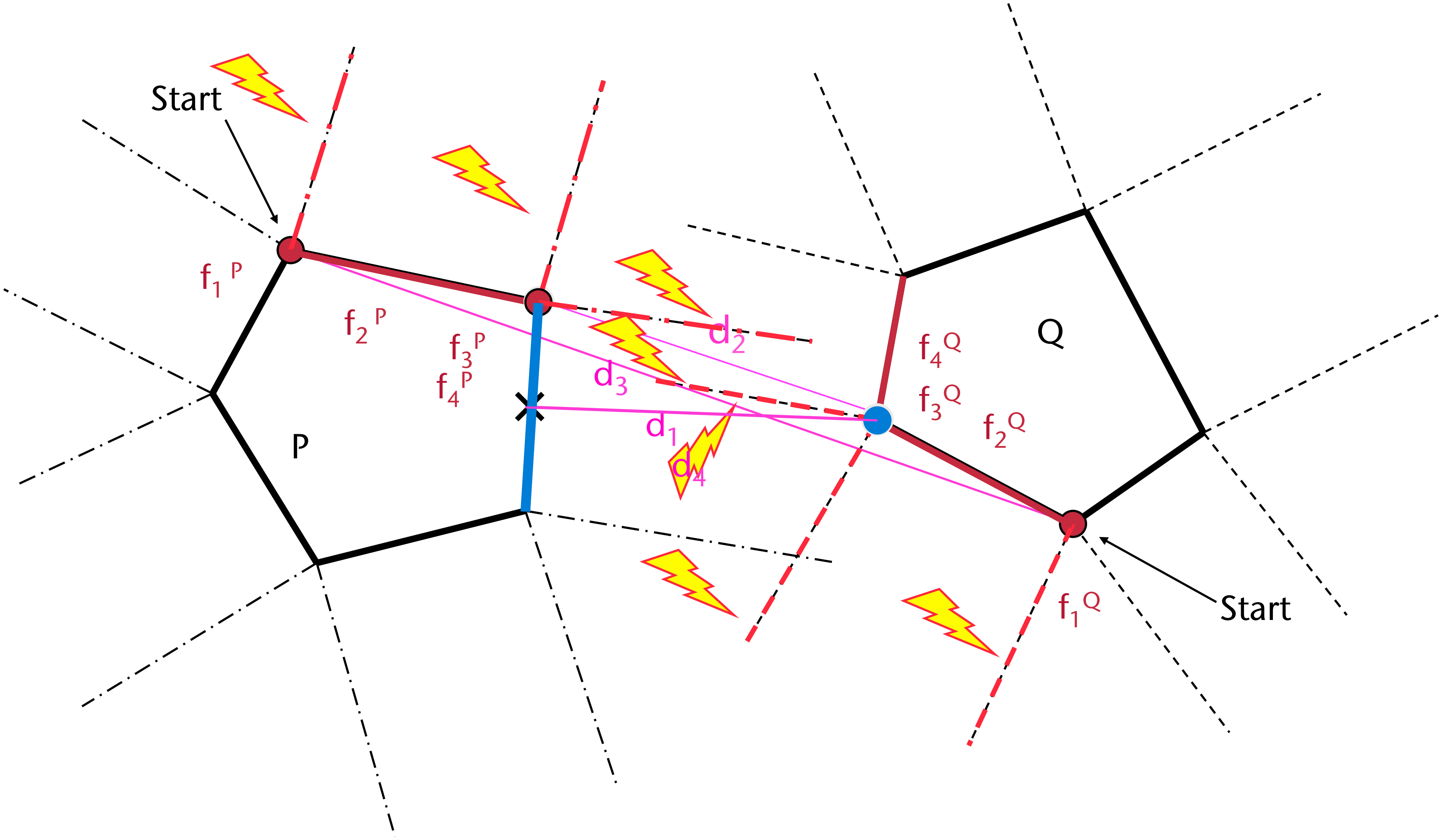
The Algorithm (Another Kind of a Steepest Descent)

```
Start with two arbitrary features $f^P$, $f^Q$ on P and Q, resp.
while ( $f^P$, $f^Q$ ) are not (yet) closest features and dist( $f^P$, $f^Q$ ) > 0 :
    if ($f^P$,$f^Q$) has been considered already:
        return “collision” (b/c we've hit a cycle)
    compute $p$ and $q$ that realize the distance between $f^P$ and $f^Q$
    if $p \in V(q)$ und $q \in V(p)$ :
        return “no collision”, ($f^P$,$f^Q$) are the closest features
    if $p$ lies on the "wrong" side of $V(q)$ :
        $f^P$ := the feature on that "other side" of $V(q)$
    do the same for $q$, if $q \notin V(p)$
if dist( $f^P$, $f^Q$ ) > 0 :
    return "no collision"
else
    return "collision"
```

Notice: in case of collision, some features are inside the other object, but we did not compute Voronoi regions inside obnjects!

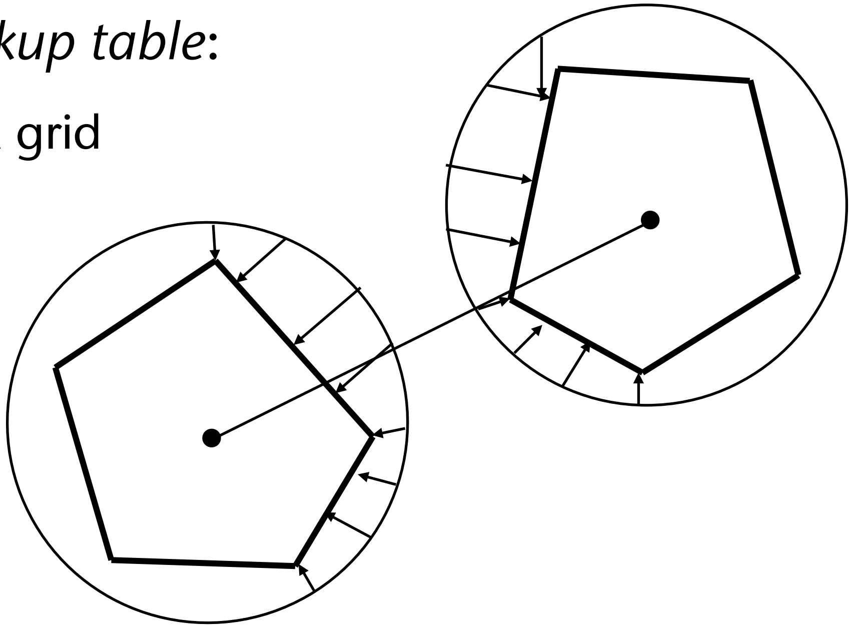
→ hence the chance for cycles

Animation of the Algorithm

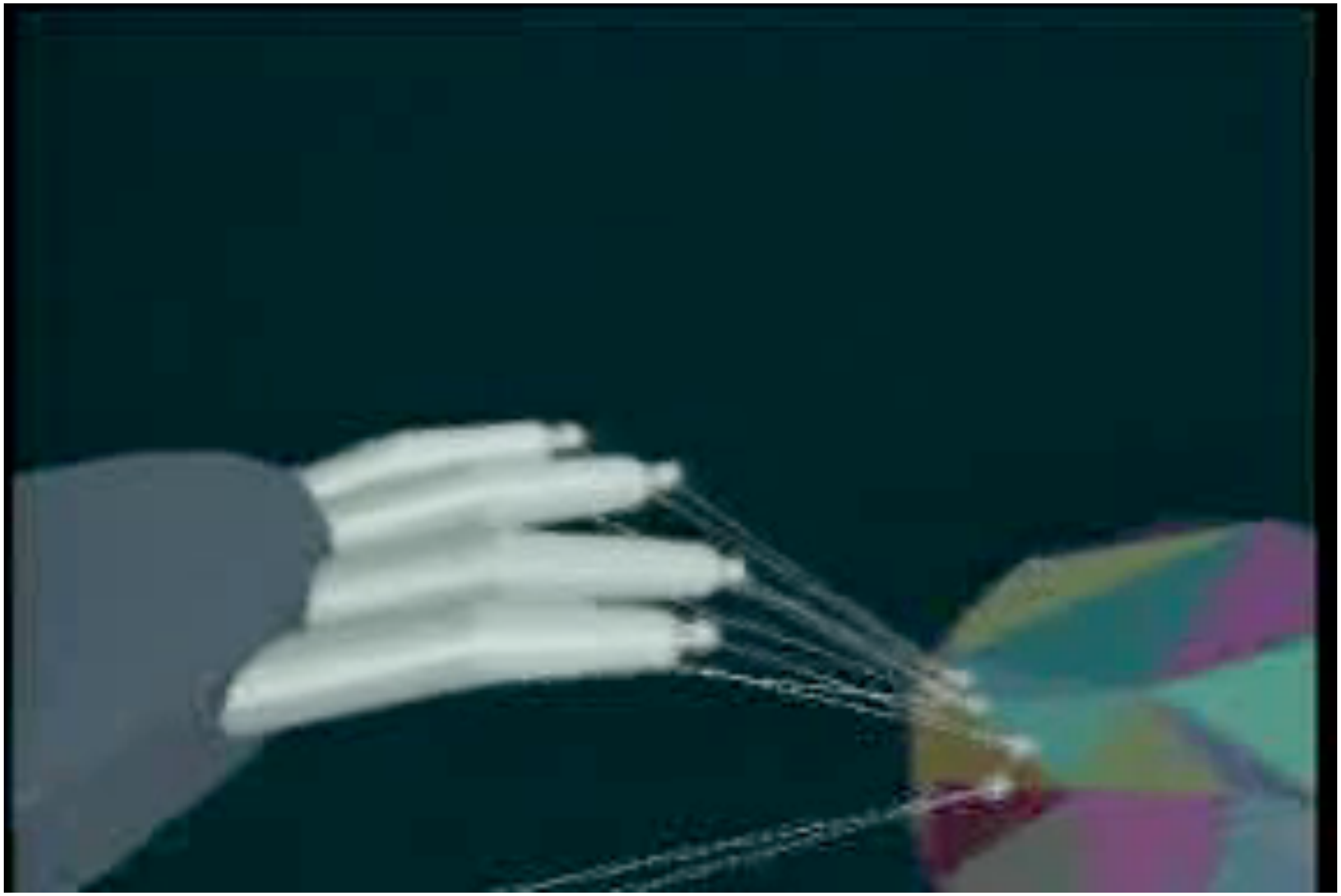


Some Remarks

- A little question to make you think:
  Actually, we don't really need the *Voronoi diagram!*
  (but with a *Voronoi diagram*, the algorithm is faster)
- The running time (in each frame) depends on the "degree" of temporal coherence
- Better initialization by using a *lookup table*:
  - Partition a surrounding sphere by a grid
  - Put each feature in each grid cell that it covers when propjected onto the sphere
  - Connect the two centers of a pair of objets by a line segment
  - Initialize the algorithm by the features hit by that line



Movie



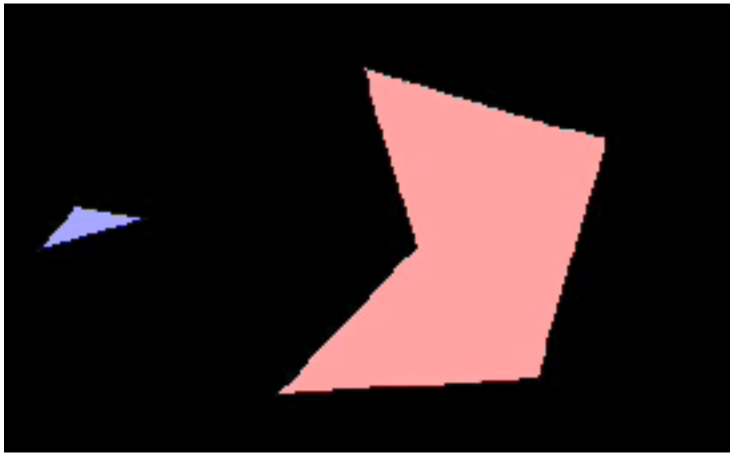
The Minkowski Sum

- Hermann Minkowski (1864 – 1909), German mathematician and physicist
- Definition (Minkowski Sum*):*
  Let *A* and *B* be subsets of a vector space;
  the Minkowski sum of *A* and *B* is defined as
  $$A \oplus B = \{\mathbf{a} + \mathbf{b} \,|\, \mathbf{a} \in A,\ \ \mathbf{b} \in B\}$$
- Analogously, we define the Minkowski difference:
  $$A \ominus B = \{\mathbf{a} - \mathbf{b} \,|\, \mathbf{a} \in A,\ \ \mathbf{b} \in B\}$$
- Clearly, the connection between Minkowski sum and difference:
  $$A \ominus B = A \oplus (-B)$$
- Applications: computer graphics, computer vision, linear optimization, path planning in robotics, …

Some Simple Properties

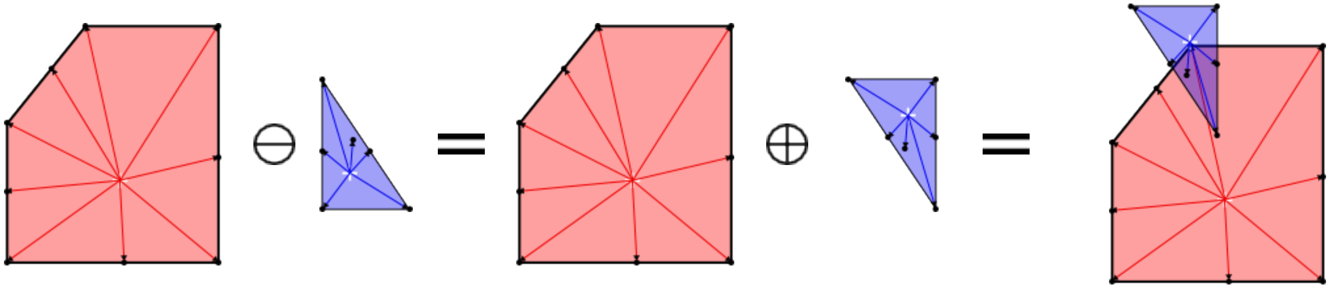
- Commutative: $A \oplus B = B \oplus A$
- Associative: $A \oplus (B \oplus C) = (A \oplus B) \oplus C$
- Distributive w.r.t. set union: $A \oplus (B \cup C) = (A \cup B) \oplus (A \cup C)$
- Invariant against translation: $T(A) \oplus B = T(A \oplus B)$

- Intuitive "computation" of the Minkowski sum/difference:

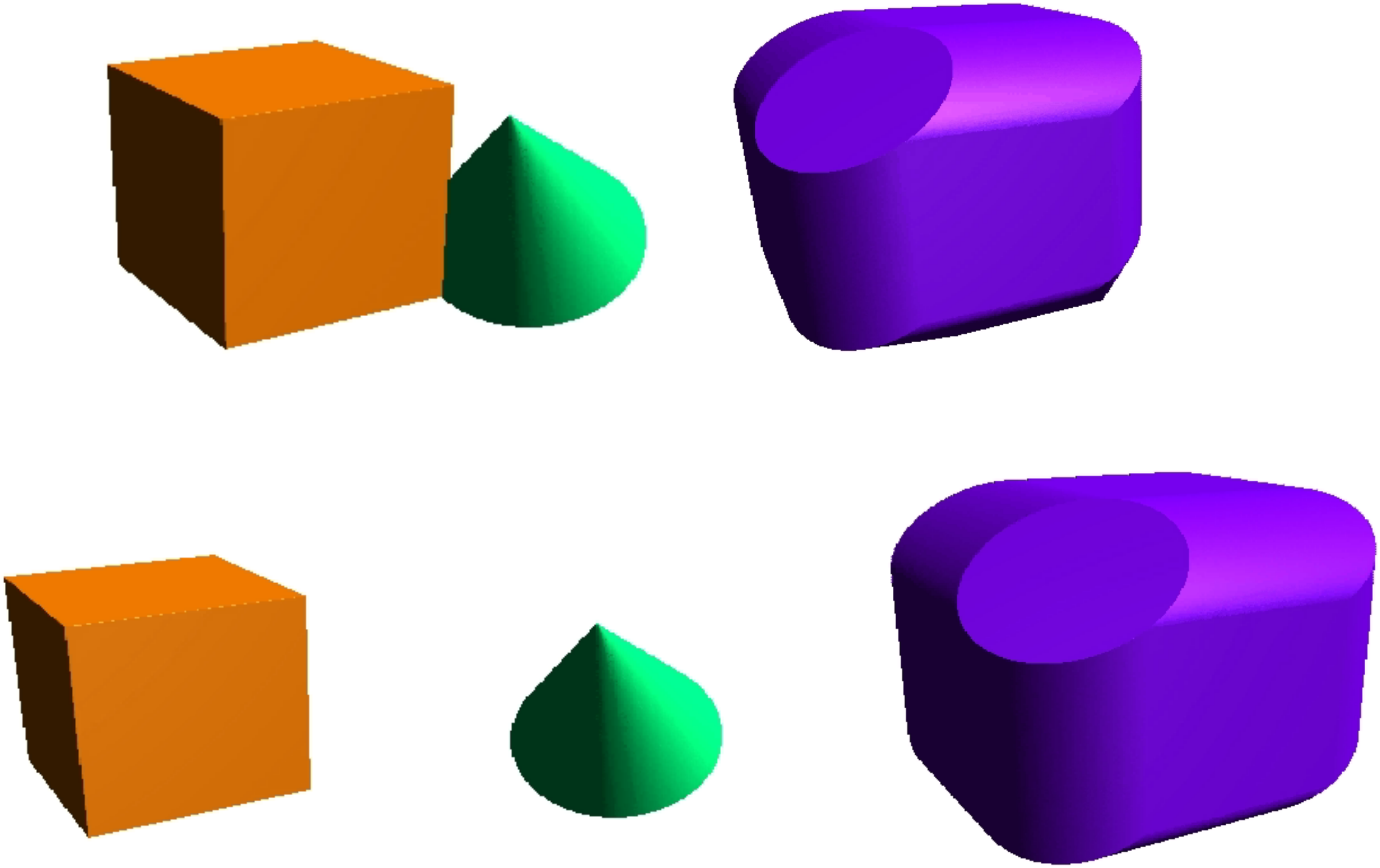


  - Warning: the yellow polygon in the animation shows the Minkowsi sum **modulo**(!) possible translations!

$$A \ominus B = A \oplus -B = C$$



Visualizations of a Simple Example



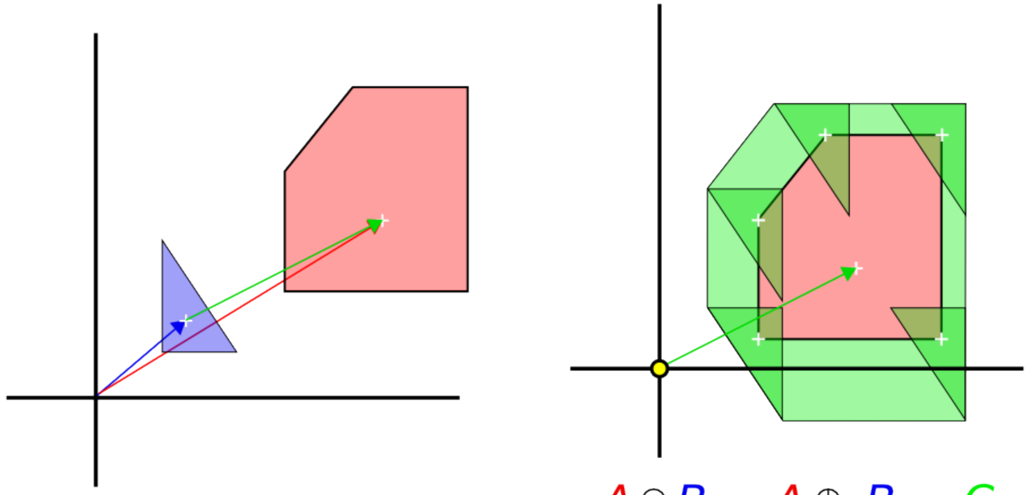
The Complexity of the Minkowski Sum (in 2D)

- Let *A* and *B* be polygons with *n* and *m* vertices, resp.:
  - If both *A* and *B* are convex,then $A \oplus B$ is convex, too, and has complexity $O(m+n)$
  - If only *B* is convex, then $A \oplus B$ has complexity $O(mn)$
  - If neither is convex, then $A \oplus B$ has complexity $O(m^2 n^2)$
- Algorithmic complexity of the computation of $A \oplus B$:
  - If *A* and *B* are convex, then $A \oplus B$ can be computed in time $O(m+n)$
  - If only *B* is convex, then $A \oplus B$ can be computed in randomized time $O(mn\, log^2(mn))$
  - If neither is convex, then $A \oplus B$ can be computed in time $O(mn^2\, log(mn))$

An Intersection Test for Two Convex Objects using Minkowski Sums

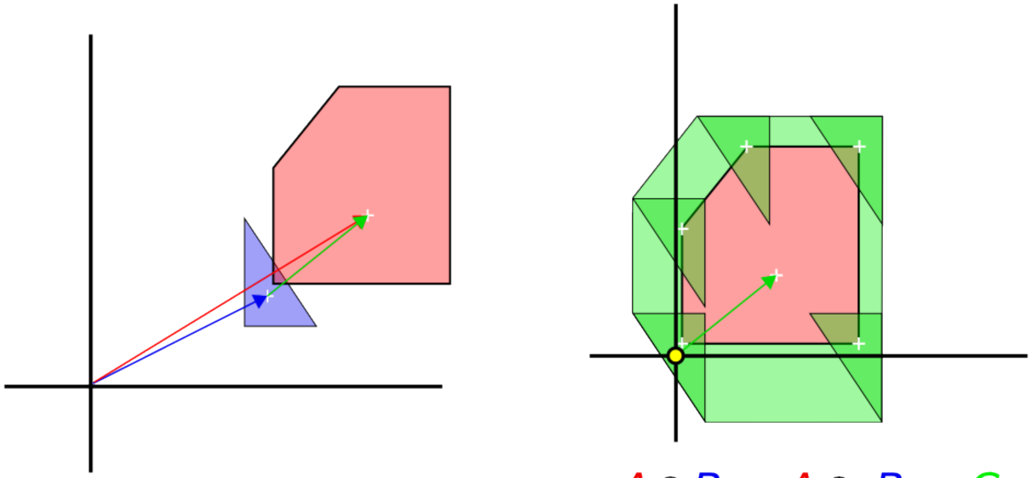
- Translate both objects so that the coordinate system's origin 0 is inside *B*
- Compute the Minkowski difference
- A and B intersect ⇔

$$0 \in A \ominus B$$



$A \ominus B = A \oplus -B = C$

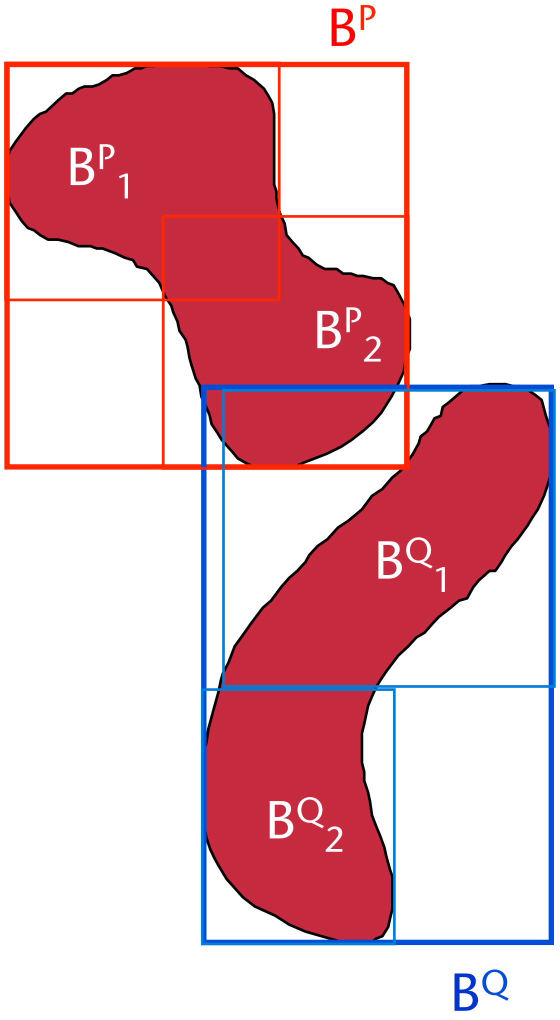
- Example where an intersection occurs:



$A \ominus B = A \oplus -B = C$

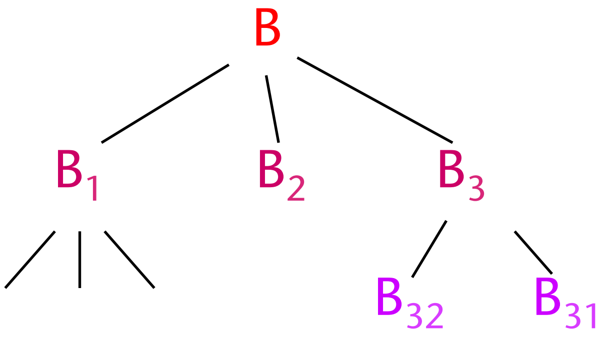
Hierarchical Collision Detection

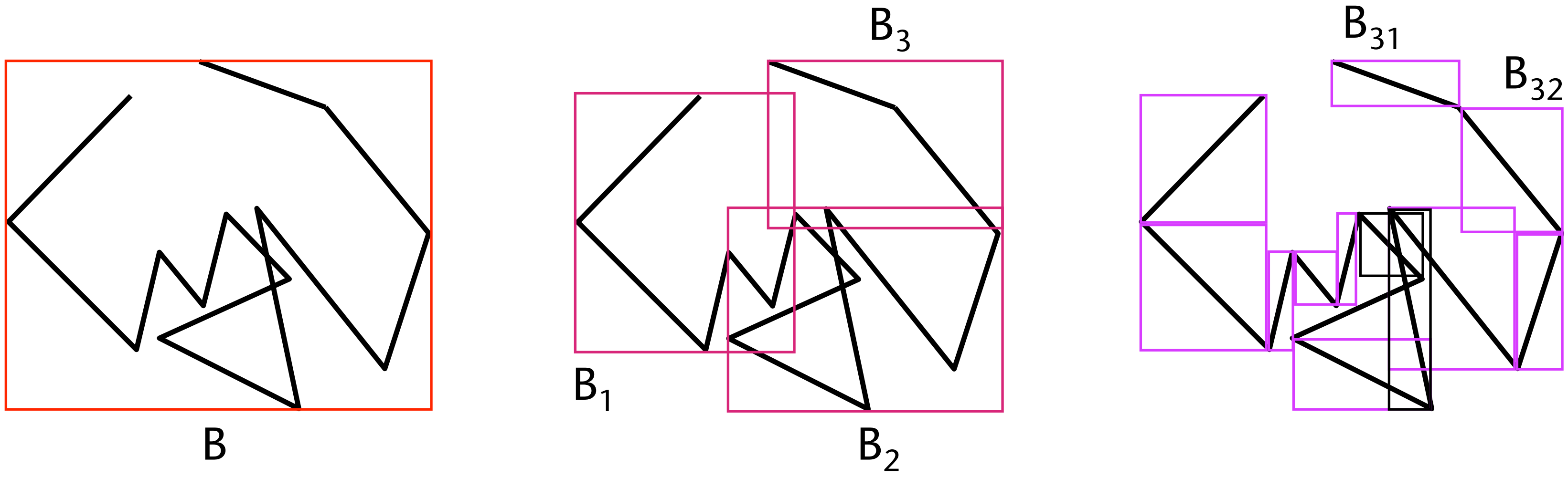
- *The* standard approach for "polygon soups"
- Algorithmic technique:
  divide & conquer



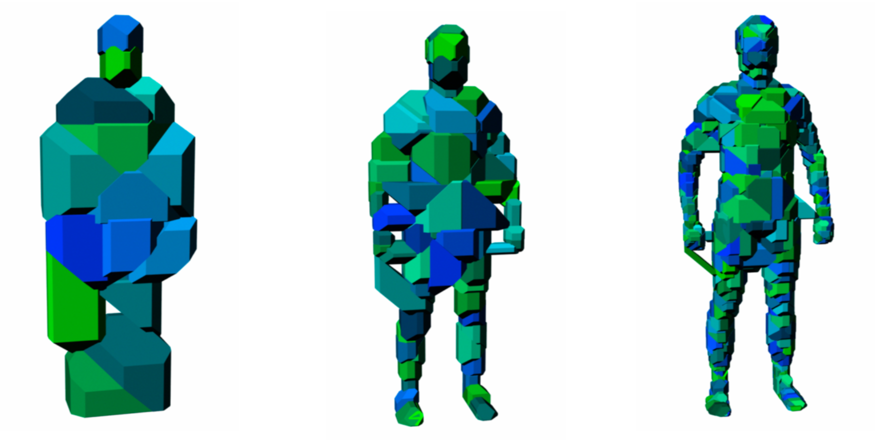
The Bounding Volume Hierarchy (BVH)

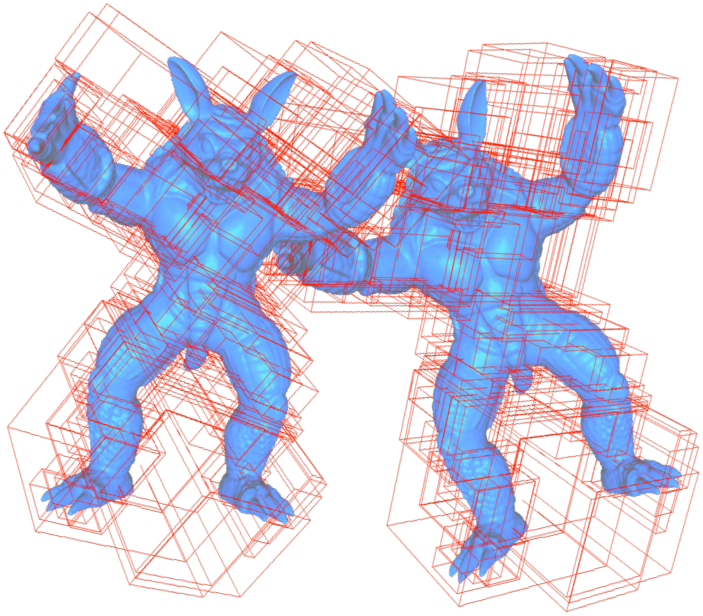
- Constructive defintion of a bounding volume hierarchy:
  1. Enclose all polygons, *P*, in a bounding volume BV(*P*)
  2. Partition *P* into subsets $P_1, \dots, P_n$
  3. Rekursively construct a BVH for each P_i and put them as children of *P* in the tree
- Typical arity = 2 or 4

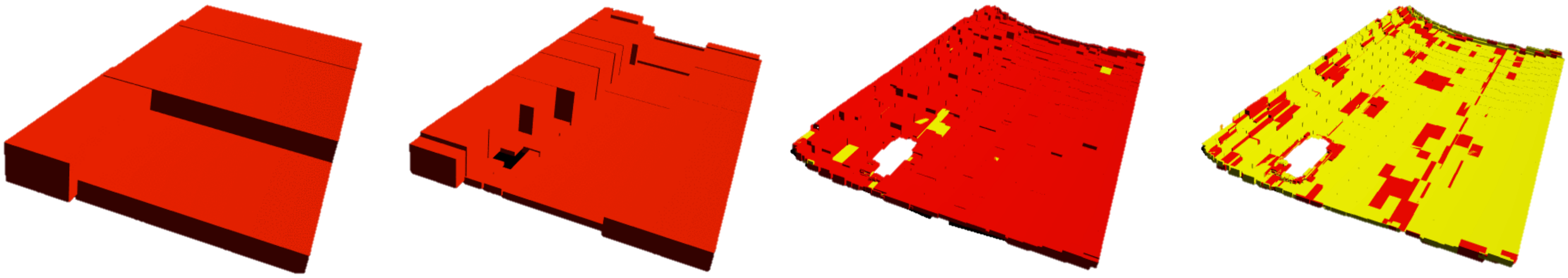




- Visualizations of different levels of some BVHs:



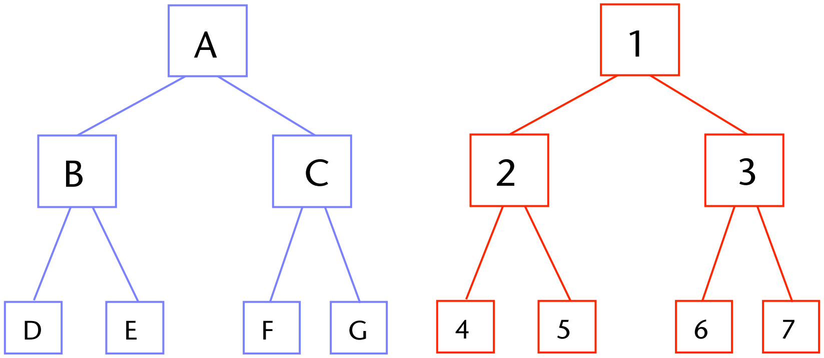


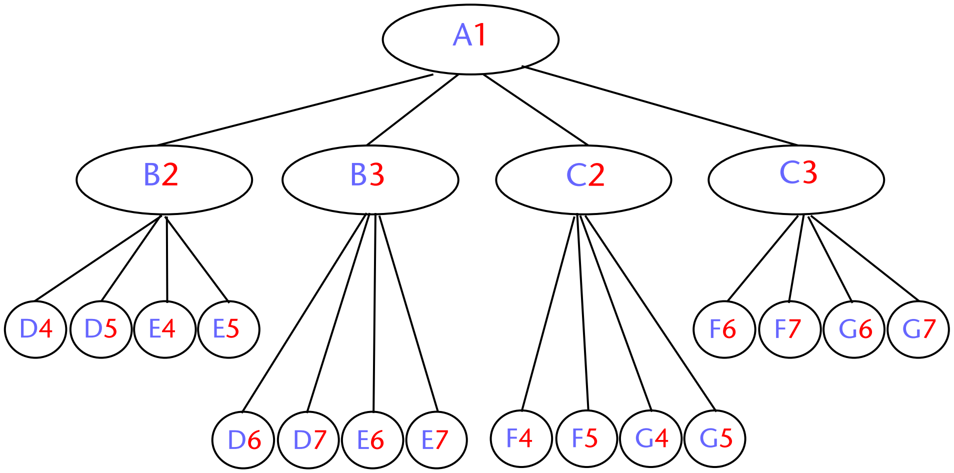


The General Hierarchical Collision Detection Algo

- Simultaneous traversal of two BVHs:

```
traverse( X, Y )
if X,Y do not overlap then
    return
if X,Y are leaves then
    check polygons
else
    for all children pairs do
    traverse( $X_i$, $Y_j$ )
```

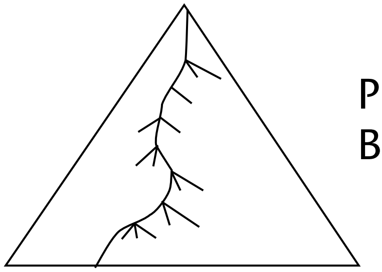




Bounding Volume Test Tree (BVTT)

A Simple Running Time Estimation

- *Best-case*: $O(\log n)$



Path through the
Bounding Volume Test Tree (BVTT)

- Extremely simple *average-case* estimation:
  - Let P[*k*] = probability that *exactly* *k* children pairs overlap, $k \in [0,\ldots,4]$

$$P[k] = \binom{4}{k} / 16 \ , \quad P[0] = \frac{1}{16}$$

  - Assumption: all events are equally likely → 16 possible events
  - Expected running time:

$$T(n) = \tfrac{1}{16} \cdot 0 + \tfrac{4}{16} \cdot T(\tfrac{n}{2}) + \tfrac{6}{16} \cdot 2T(\tfrac{n}{2}) + \tfrac{4}{16} \cdot 3T(\tfrac{n}{2}) + \tfrac{1}{16} \cdot 4T(\tfrac{n}{2})$$

$$T(n) = 2T(\tfrac{n}{2}) \in O(n)$$

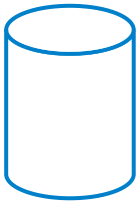
- In praxi: running time is better/worse depending on degree of overlap

Different Kinds of Bounding Volumes

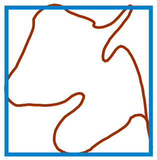
Requirements (for collision detection):

- *Very* fast overlap test → "simple" BVs
  - Even if BVs have been translated/rotated
- Little overlap among BVs on the same level in a BVH (i.e., if you want to cover the whole space with the BVs, there should be as little overlap as possible) → "*tight BVs*"

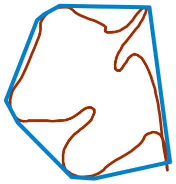
Different Kinds of Bounding Volumes



Cylinder
[Weghorst et al., 1985]

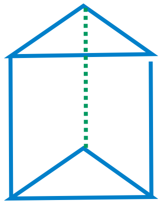


Box, AABB (R*-trees)
[Beckmann, Kriegel, et al., 1990]

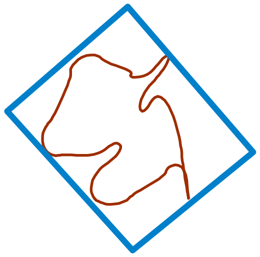


Convex hull
[Lin et. al., 2001]

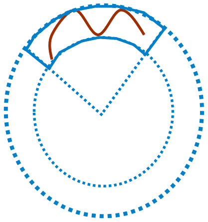
Sphere
[Hubbard, 1996]



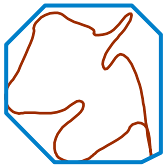
Prism
[Barequet, et al., 1996]



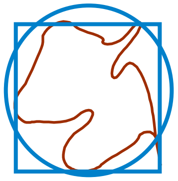
OBB (oriented bounding box)
[Gottschalk, et al., 1996]



Spherical shell
[Manocha, 1997]



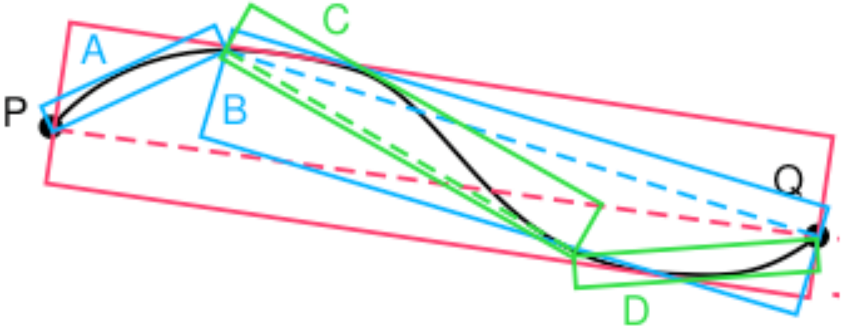
k-DOP / Slabs
[Zachmann, 1998]



Intersection of
several BVs

The Wheel of Re-Invention

- OBB-Trees: have been proposed already in 1981 by Dana Ballard for bounding 2D curves, except they called it "strip trees"



- AABB hierarchies: have been invented(?) in the 80-ies in the spatial data bases community, except they call them "R-tree", or "R*-tree", or "X-tree", etc.

Digression: the Wheel of Fortune (Rad der Fortuna)

Boccaccio De Casibus Virorum Illustrium Paris: 1467

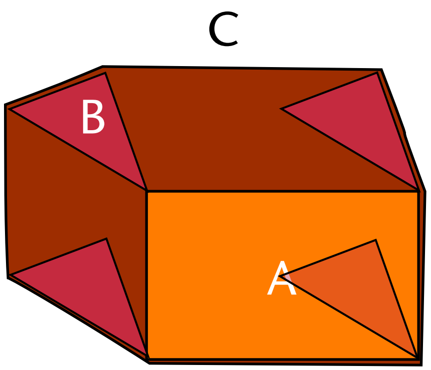
Codex Buranus

The Intersection Test for Oriented Bounding Boxes (OBB)

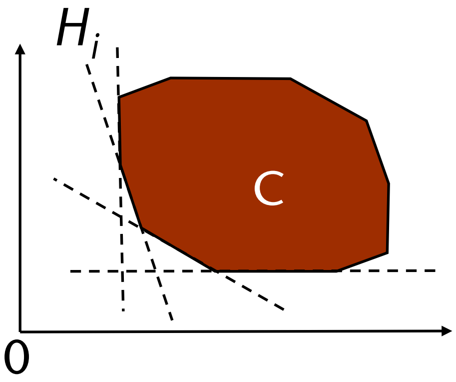
- Lemma "Separating Axis Test" (SAT):
  Let *A* and *B* be two convex 3D polyhedra.
  If there is a separating plane, then there is also a separting plane that is either parallel to one side of *A*, or parallel to one side of *B*, or parallel to one edge of *A* and one edge of *B* simultaneously.
  [Gottschalk, Lin, Manocha; 1996]

- The "separating plane" lemma
  (just a different wording of the "separating axis" lemma):
  Two convex polyhedra *A* and *B* do *not* overlap ⇔
  there is an axis (line) in space so that the projections of *A* and *B* onto that axis do not overlap.
  This axis is called the separating axis.

Proof of the SAT Lemma



1. Assumption: A and B are disjoint
2. Consider the Minkowski sum $C = A \ominus B$
3. All faces of C are either parallel to one face of A, or to one face of B, or to one edge of A *and* one of B (the latter cannot be seen in 2D)
4. C is convex
5. Therefore: $C = \bigcap_{i=1}^{m} H_i$
6. $A \cap B = \emptyset \Leftrightarrow 0 \notin C$
7. $\exists i : 0 \notin H_i$ (i.e., 0 is outside some H_i)
8. That H_i defines the separating plane; the line perpendicular to H_i is the separating axis.



Actually Computing the SAT for OBBs

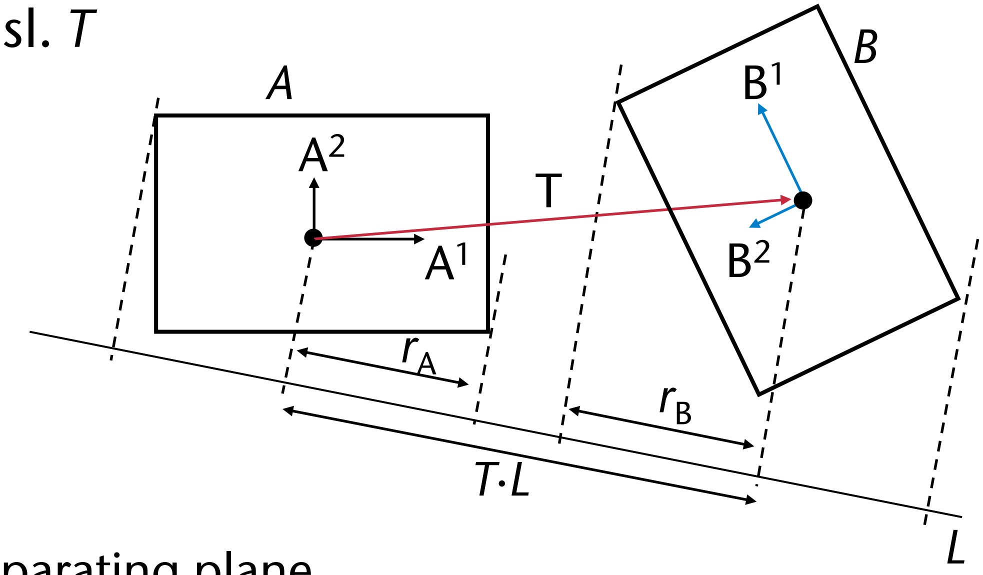
- W.l.o.g.: compute everything in the coordinate frame of OBB *A*
- *A* is defined by: center *c*, axes A^1, A^2, A^3 , and extents a^1, a^2, a^3, resp.
- B's position relative to *A*

  is defined by rot. *R* and transl. *T*
- In the coord. frame of *A*:

  B^i are the columns of *R*
- Let L be a line in space;

  then *A* and *B* overlap,

  if $|T \cdot L| < r_A + r_B$
  - Remark: *L* = normal to the separating plane

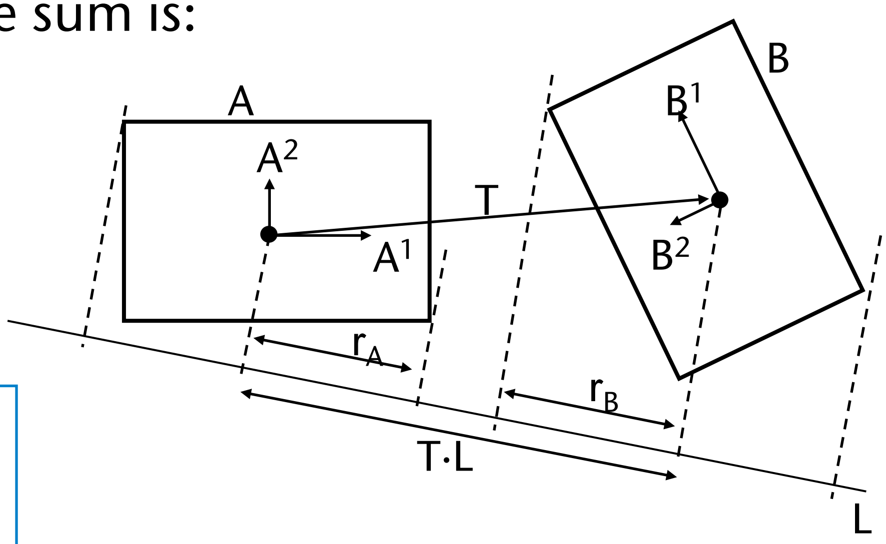


- According to the lemma, we need to check only a few special lines
- With boxes, that number of special lines = 15

- Example: $L = A^1 \times B^2$
- We need to compute: $r_A = \sum_i a_i |A^i \cdot L|$ (and similarly r_B)
- For instance, the 2nd term of the sum is:

$$\begin{aligned} & a_2 A^2 \cdot (A^1 \times B^2) \\ = & a_2 B^2 \cdot (A^2 \times A^1) \\ = & a_2 B^2 \cdot A^3 \\ = & a_2 R_{32} \end{aligned}$$

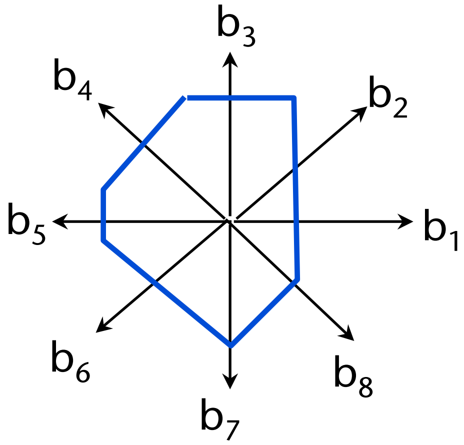
Since we compute everything in A's coord. frame
→ A^3 is 3rd unit vector, and
B^2 is 2ns column of R



- In general, we have one test of the following form for each of the 15 axes:

$$|T \cdot L| < a_2|R_{32}| + a_3|R_{22}| + b_1|R_{13}| + b_3|R_{11}|$$

Discretely Oriented Polytopes (*k*-DOPs)



- Definition of *k*-DOPs:

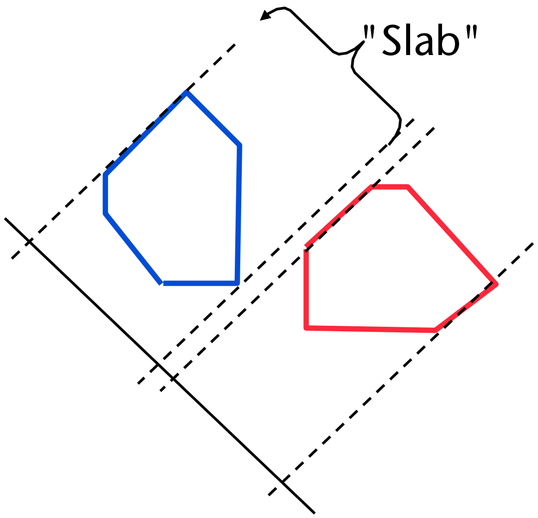
  Choose *k* fixed vectors $\mathbf{b}_i \in \mathbb{R}^3$, with *k* even,

  and $b_i = - b_{i+k/2}$.

  A *k*-DOP is a volume defined by

$$D = \bigcap_{i=1..k} H_i \quad , \quad H_i : \mathbf{b}_i \cdot x - d_i \leq 0$$

- A *k*-DOP is completely described by: $D = (d_1 \ldots d_k) \in \mathbb{R}^k$
- The overlap test for two (axis-aligned) *k*-DOPs:



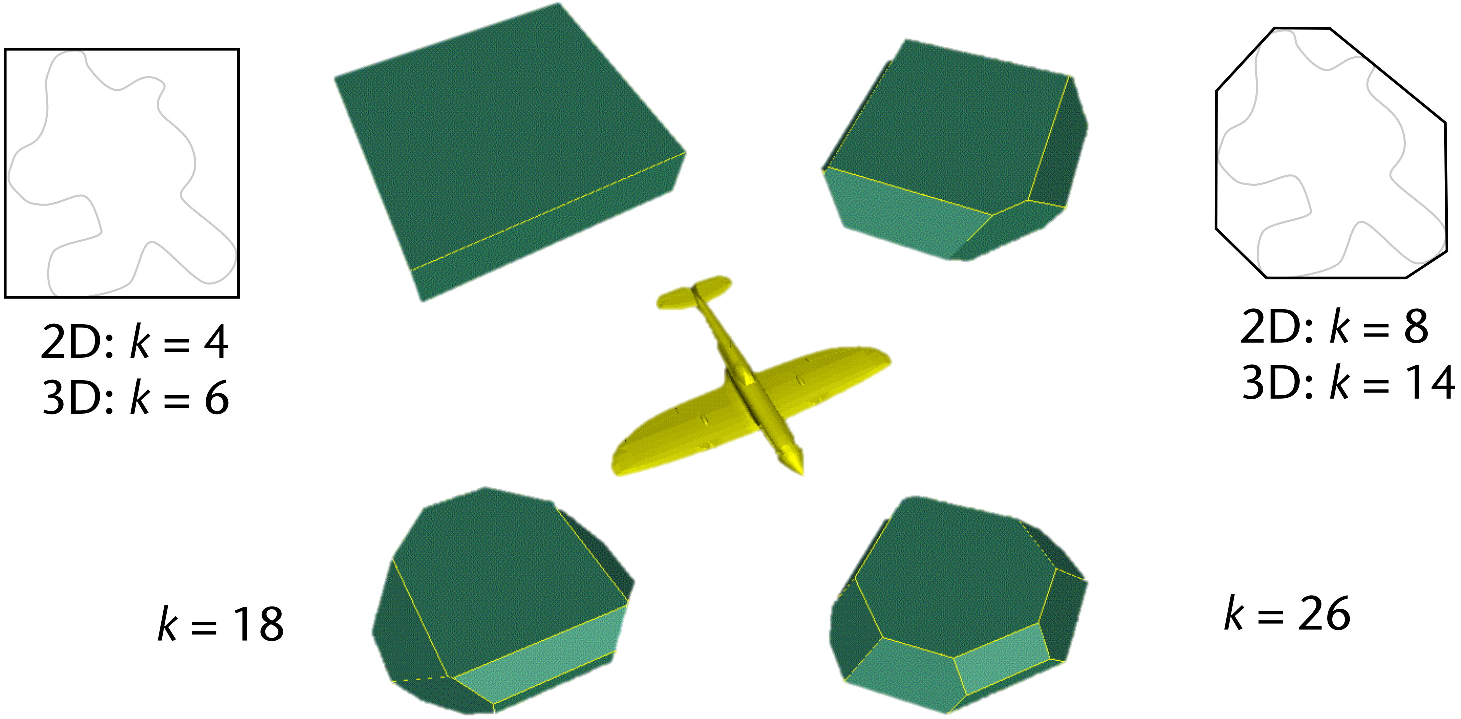
$$D^1 \cap D^2 = \varnothing \Leftrightarrow$$

$$\forall i = 1, .., \frac{k}{2} : \left[d_i^1, d_{i+\frac{k}{2}}^1 \right] \cap \left[d_i^2, d_{i+\frac{k}{2}}^2 \right] = \varnothing$$

i.e., it's just *k*/2 interval tests

Some Properties of *k*-DOPs

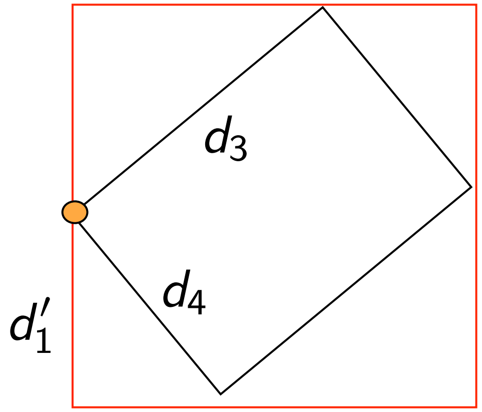
- AABBs are special DOPs
- The overlap test takes time $\in O(k)$, k = number of orientations
- With growing *k*, the convex hull can be approximated arbitrarily precise:



The Overlap Test for Rotated *k*-DOPs

- The idea: enclose an "oriented" DOP by a new axis-aligned one:
  - The object's orientation is given by rotation *R* & translation *T*
  - The axis-aligned DOP D' = ($d'_1, \ldots, d'_k$) can be computed as follows (without proof):

$$d'_i = \boxed{\mathbf{b}_i \begin{pmatrix}\mathbf{c}_{j^i_1}\\ \mathbf{c}_{j^i_2}\\ \mathbf{c}_{j^i_3}\end{pmatrix}^{-1}} \begin{pmatrix} d_{j^i_1}\\ d_{j^i_2}\\ d_{j^i_3}\end{pmatrix} + \boxed{\mathbf{b}_i T},$$

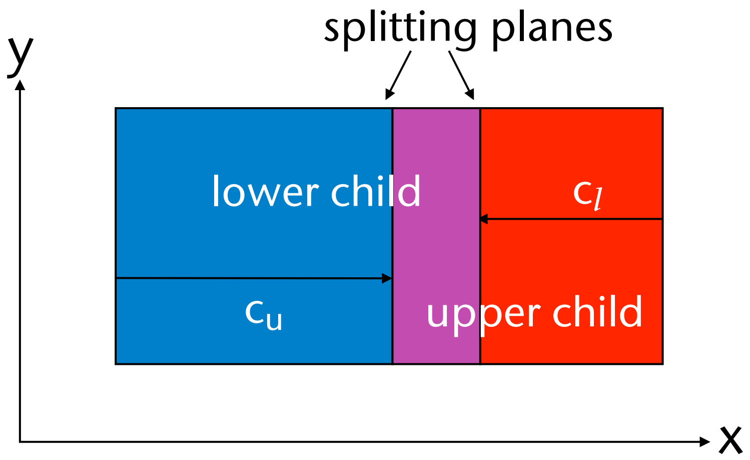


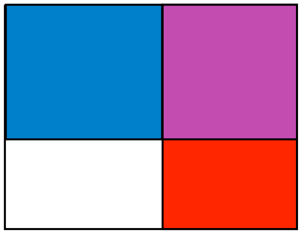
with $\mathbf{c}_j = \mathbf{b}_j R^{-1}$

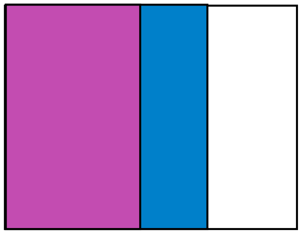
  - The correspondence $j^i{}_l$ is identical for all DOPs in the same hierarchy (thus, it can be precomputed)
  - Complexity: O(*k*)
    - Compare this to a SAT-based overlap test

Restricted Boxtrees (a Variant of kd-Trees)

- Restricted Boxtrees are a combination of kd-trees and AABB trees:
  - The idea: for the left child of a node B, split off a portion of the "right" part of the box B; for the right child of B, split off a portion of the left part of B
- Memory usage: 1 float, 1 axis ID, 1 pointer (= 9 bytes)
- Other names for the same DS:
  - Bounding Interval Hierarchy (BIH)
  - Spatial kd-tree (SKD-Tree)

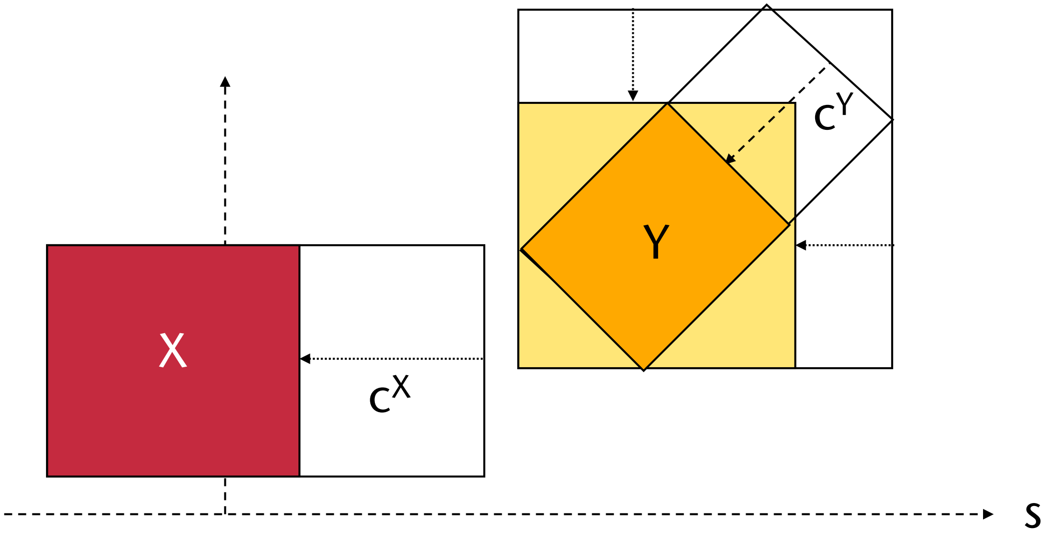






- Overlap Tests by "re-alinment" (i.e., enclosing the non-axis-aligned box in an axis-aligned one, exploiting the special structure of restricted boxtrees):

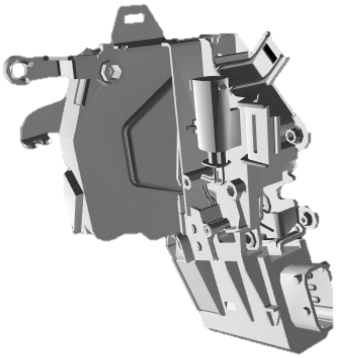
  12 FLOPs (8.5 with a little trick)



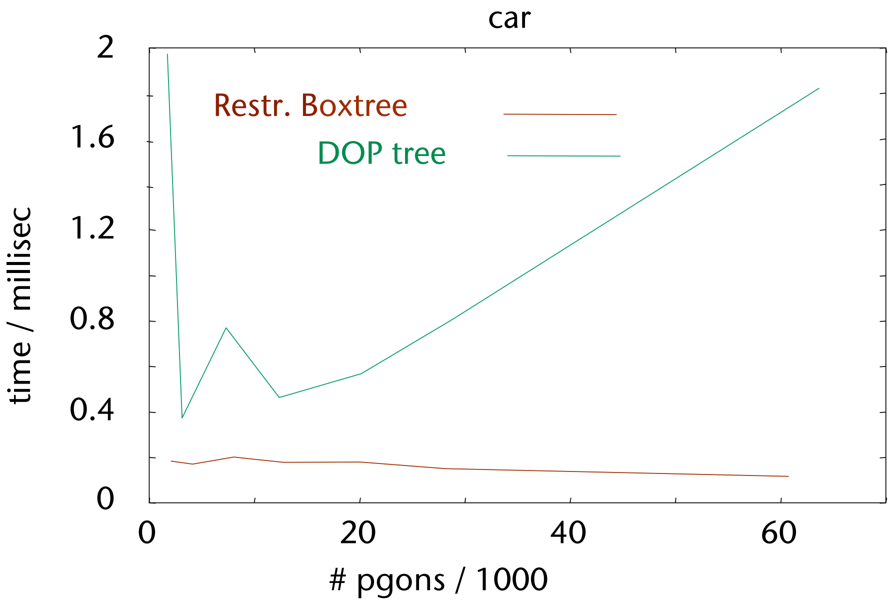
- Compare this to
  - SAT: 82 FLOPs
  - SAT lite: 24 FLOPs
  - Sphere test: 29 FLOPs

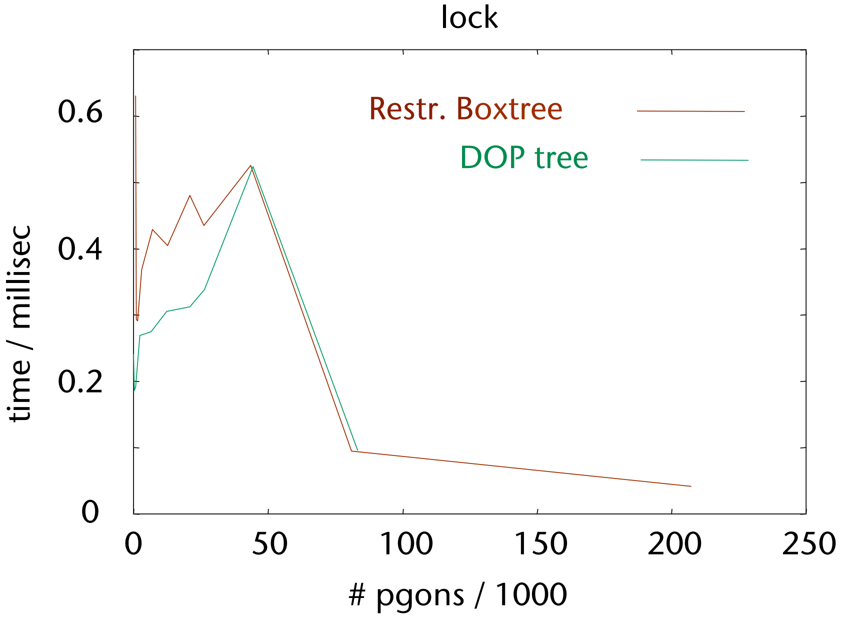
Performance

Car (courtesy VW)



Door lock (BMW)





The Construction of BV Hierarchies

- Obviously:
  if the BVH is bad → collision detection has a bad performance
- The general algorithm for BVH construction: *top-down*
  1. Compute the BV enclosing the set of polygons
  2. Partition the set of polygons
  3. Recursively compute a BVH for each subset
- The essential question: the splitting criteria?
- Guiding principle: the expected cost of collision detection incured by a particular split

$$C(X,Y) = 4 + \sum_{i,j=1,2} P(X_i,Y_j)\,C(X_i,Y_j)$$
$$\approx 4\left(1 + P(X_1,Y_1) + \cdots + P(X_2,Y_2)\right)$$

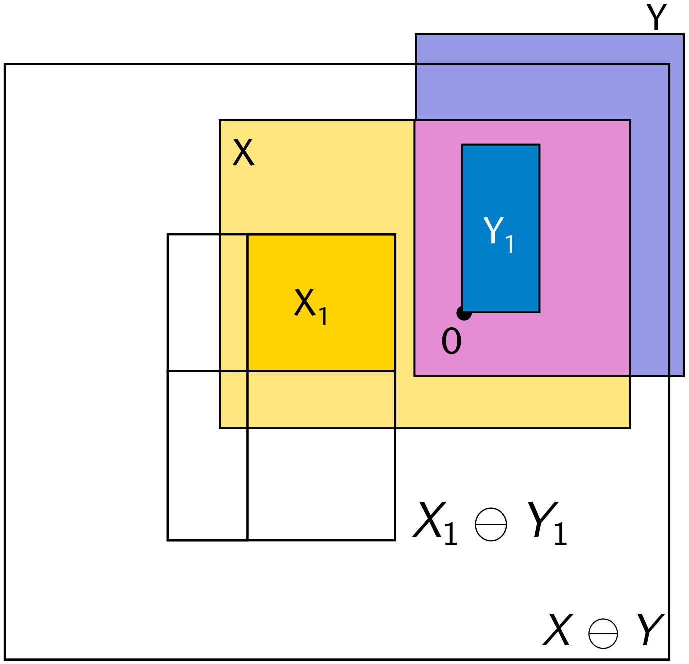
- Goal: estimation of P(X_i,Y_j)
- Our tool: the Minkowski sum
- Reminder:

$$X_i \cap Y_j = \varnothing \;\Leftrightarrow\; 0 \notin X_i \ominus Y_j$$

- Therefore, the probability is:

$$P(X_i, Y_j) = \frac{\#\ \text{``good'' cases}}{\#\ \text{all possible cases}}$$

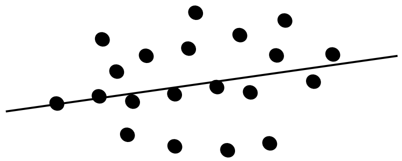
$$= \frac{\text{vol}(X_i \ominus Y_j)}{\text{vol}(X \ominus Y)} = \frac{\text{vol}(X_i \oplus Y_j)}{\text{vol}(X \oplus Y)} \approx \frac{\text{vol}(X_i) + \text{vol}(Y_j)}{\text{vol}(X) + \text{vol}(Y)}$$



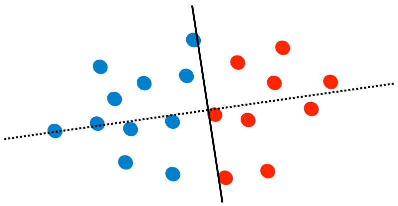
- Conclusion: for a good BVH (for coll.det.) minimize the total volume of the children of each node

Usual Algorithm for Constructing a BVH

1. Find good orientation for a "good" splitting plane using PCA



2. Find the minimum of the total volume by a sweep of the splitting plane along that axis



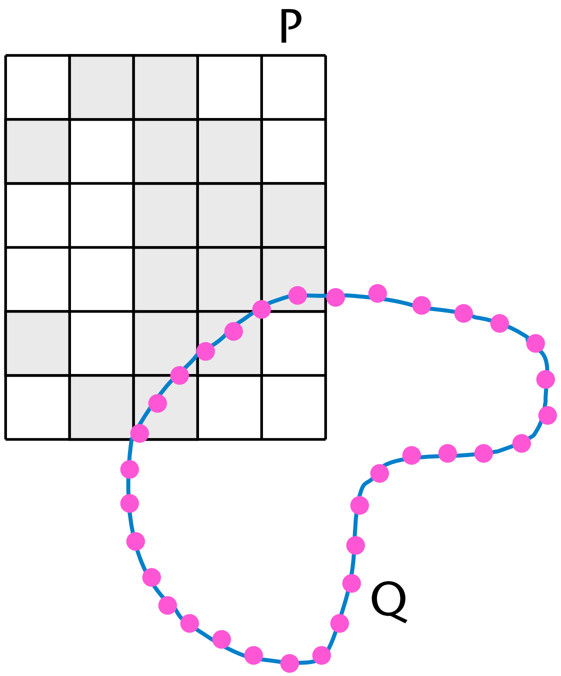
- Complexity of that *plane-sweep* algorithm:

$$T(n) = n \log n + T(\alpha n) + T((1-\alpha)n) \in O\left(n \log^2 n\right)$$

  - Assumption: splits (α) are not too uneven

Coll.Det. Algorithm Depends on Object Representation

- Example: Voxmap-Pointshell
  - Objects are represented by point shell and by a voxel grid
- The fundamental operation:
  does a point hit a black voxel?
- Problems:
  - What to do in case of non-closed objects?
  - Memory consumption for all the voxels!
    - Hierarchy might help, but also slows coll.det. down
  - Collision detection is not exact (b/c of discretization)



Collision Detection among Morphing Objects

- Definition of *Morphing*:

  Given n objects O^i (called morph targets)

  with vertices v^i_j and weights w_i , $\sum_i w_i = 1$.

  Then the morphed object is given by the vertices:

$$\bar{v}_j = \sum_{i=1}^{n} w_i v^i_j \quad , \; j = 1, \ldots, N$$

- Alternative representation:
  - Represent objects O^i as a single, long "vertex vector": $\mathbf{v}^i = \begin{pmatrix} v^i_{1,x} \\ v^i_{1,y} \\ v^i_{1,z} \\ v^i_{2,x} \\ \vdots \\ v^i_{N,z} \end{pmatrix}$
  - Then, the morphed object is:

$$\bar{\mathbf{v}} = \sum_{i=1}^{n} w_i \mathbf{v}^i$$

- Note: all meshes must have the same "topolgy" (i.e., connectivity)!

- *Morphing* of k-DOP's:

  Given n DOPs $D^i = \left(s_1^i, \ldots, s_{\frac{k}{2}}^i, e_1^i, \ldots, e_{\frac{k}{2}}^i\right)$.

  We define the morphed DOP

$$\overline{D} = (\overline{s}_1, \ldots, \overline{s}_{\frac{k}{2}}, \overline{e}_1, \ldots, \overline{e}_{\frac{k}{2}}) \ , \ (\overline{s}_j, \overline{e}_j) = \left(\sum w_i s_j^i \ , \ \sum w_i e_j^i\right)$$

- Conjecture:

  If the morph targets O^i are bounded by the D^i, then the morphed object is bounded by the morphed DOP, i.e.

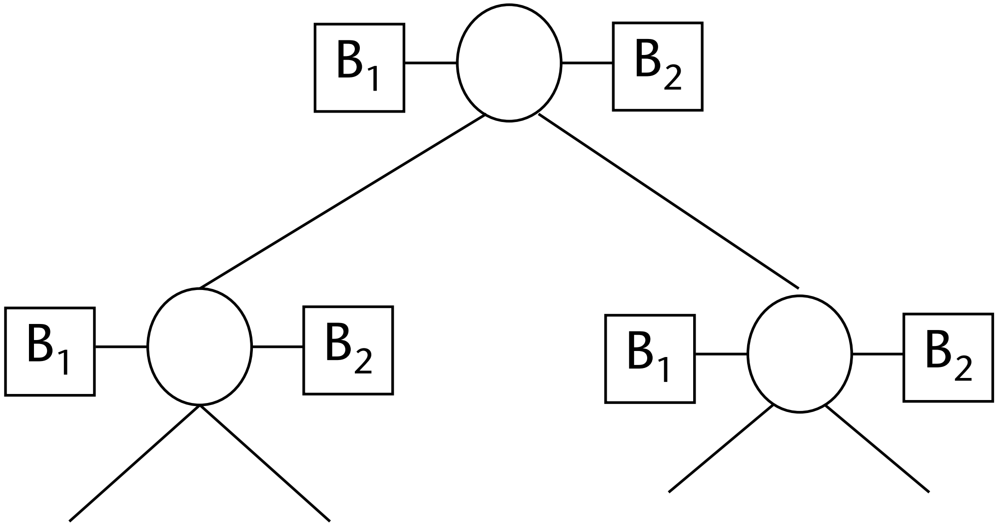
$$\forall l \ : \ \mathbf{v}_l^i \in D^i \quad \text{then} \quad \overline{\mathbf{v}}_j \in \overline{D}$$

- Proof:

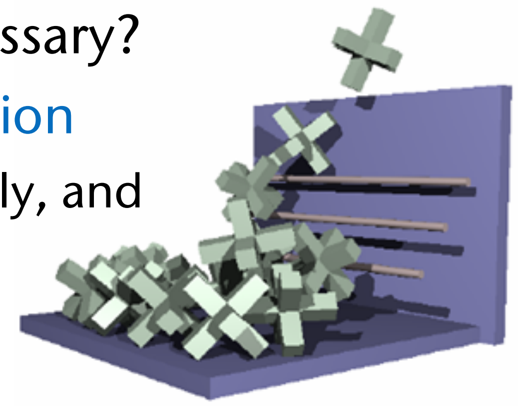
$$\forall l \ : \ \overline{s}_j = \sum_{i=1}^{n} w_i s_j^i \ \leq \ \sum_{i=1}^{n} w_i \left(\mathbf{v}_l^i \cdot \mathbf{b}^j\right) \ \leq \ \sum_{i=1}^{n} w_i e_j^i = \overline{e}_j$$

- This is also true analogously for spheres (doesn't work for OBBs)

- Data structure of a BVH for morphed objects:
  - At each node of the "morphed BVH", store a BV for each of the morph targets
  - Each of these BV's of the morph targets must enclose the same subset of polygons!

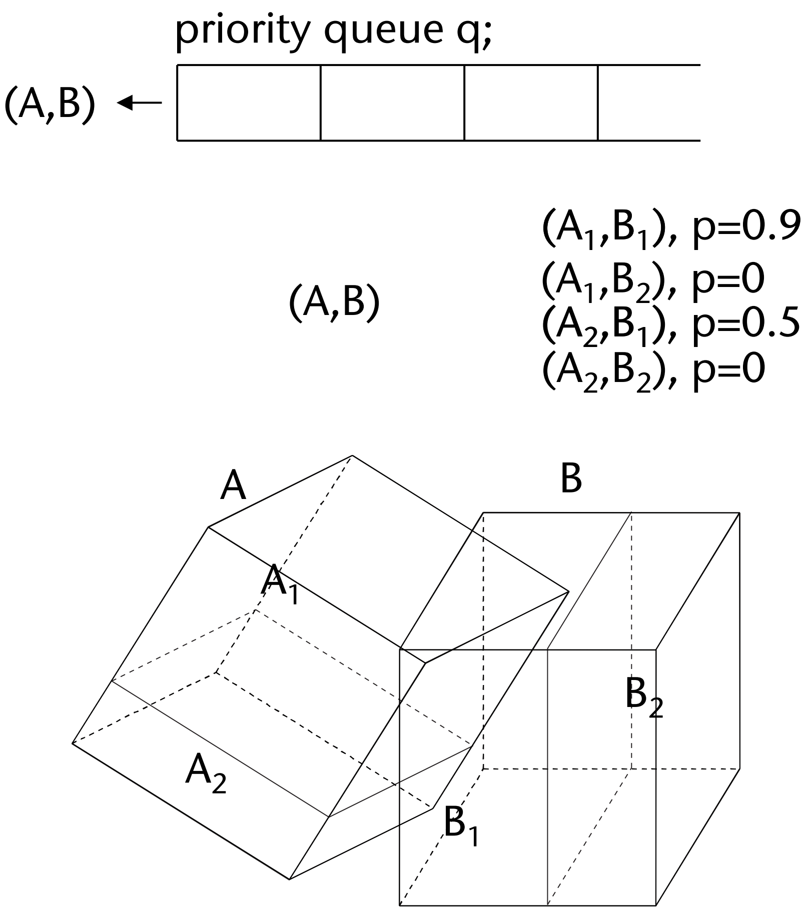


Time-Critical Collision Detection



- Is 100% exact collision detection really necessary?
- Consequence: approximate collision detection
  - Try to perform collision detection approximately, and
  - Try to take advantage of that → increase speed
- Problems of classical BVH traversal:
  - Early exit does not yield *any* information at all
  - There is no level of detail (unless specifically crafted)
- Goal: continuous and and controlled balance between running time and accuracy
- Idea: utilize a remaining degree of freedom in the simultaneous traversal algorithm
- New algorithm:
  - For a given pair of BV's, estimate the probbility of collision within
  - First "visit" those subtrees with high probability
  - No stack any more, instead use priority queue (p-queue)

Overview of the New Probability-Driven Algorithm



priority queue q;

(A,B) ←

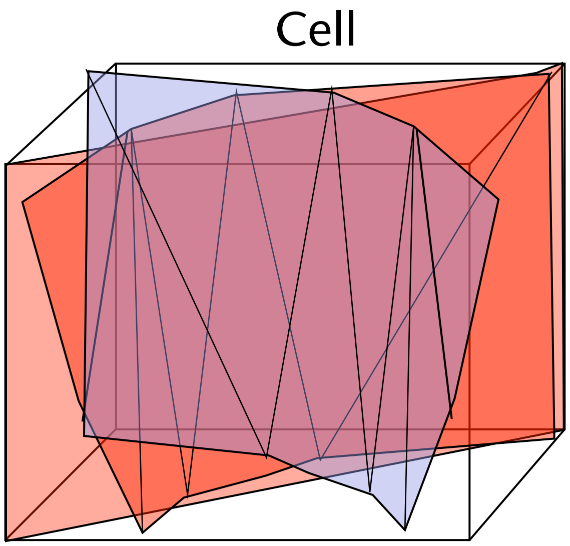
(A,B)

(A_1,B_1), p=0.9
(A_1,B_2), p=0
(A_2,B_1), p=0.5
(A_2,B_2), p=0

<u>Traverse(A,B)</u>

```
P-queue q
q.insert(A,B,1)
while q not empty
    A,B ← q.pop
    forall $A_i$, $B_j$
        p ← Pr[ collision in $A_i$, $B_j$ ]
        if p ≥ pmin
            return “collision”
        if p ≥ 0
            q.insert($A_i$, $B_j$, p)
return “no collision”
```

Thought Experiment ("Gedankenexperiment")

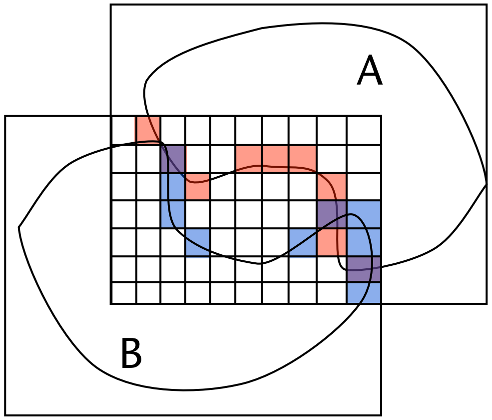


"Well-Filled" Cell

→ Collision Cell

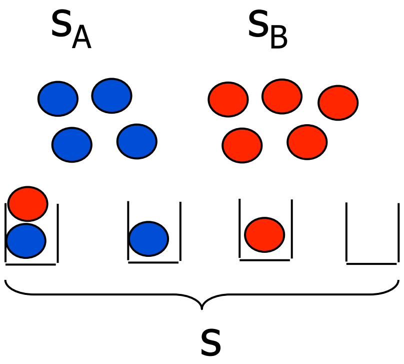
Estimation of the Probability of a Collision (Idea only)

- "Well-filled" = surface area in a cell is larger than a specific threshold
- Idea:
  - Partition $A \cap B$ by grid
  - Compute probability of cell that is *well-filled* by *A* and *B*



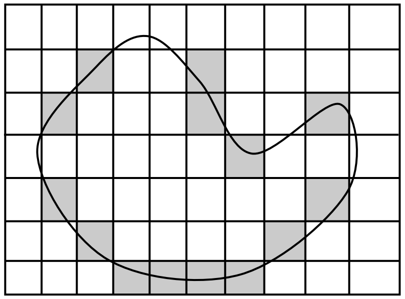
- During runtime: estimate following param's
  - s = number of grid cells in $A \cap B$
  - s_A , s_B = number of cells well-filled by surface of *A* or *B*, resp.
- Estimate probability for intersection by probability that one (or more) cell is well-filled by *A* and *B*:
  - Purely combinatoric "balls into bins" model
  - Probability

$$Pr = 1 - \frac{\binom{s-s_B}{s_A}}{\binom{s}{s_A}}$$



Efficient Implementation

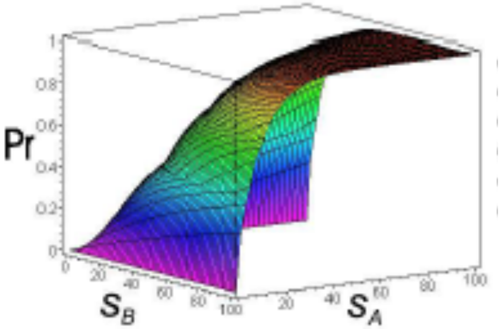
- Partitioning $A \cap B$ and counting number of well-filled cells at runtime is too expensive
- Solution: preprocessing and further estimations
- Augmented BVH (ADB-tree):
  - For each BV, partition BV by grid (e.g., 8^3)
  - Store number of well-filled grid cells with node
    - Just one additional integer per node!



- At runtime, estimate s_A and s_B by

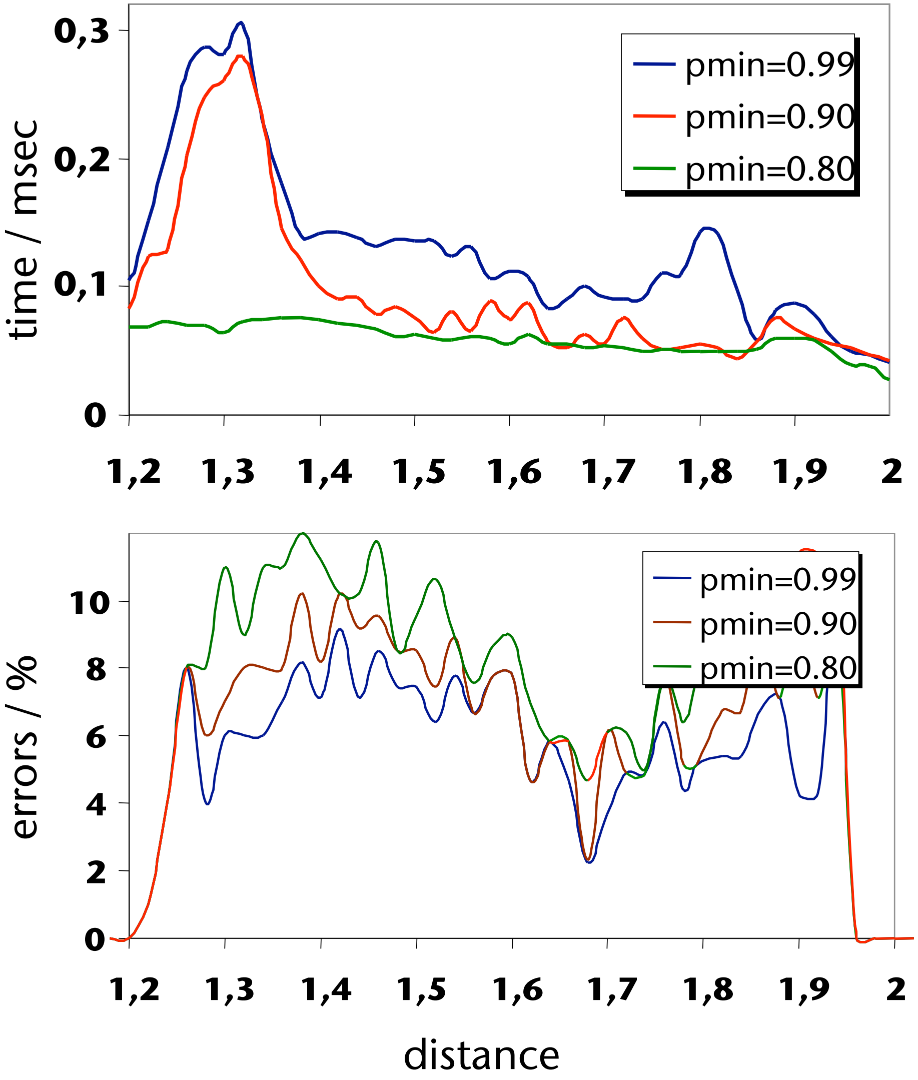
$$s'_A = s_A \frac{\mathrm{Vol}(A)}{\mathrm{Vol}(A \cap B)}$$

- Precompute function *Pr* and store in a Lookup Table



Result

- Time vs. erro:



Open Problems

- Can we estimate collision normals that way, too?
- Utilize orientation of polygons, in order to improve the estimation of an intersection!
- What about deformable geometry?!